

GOLDEN VALLEY HIGH SCHOOL
AP[®] CALCULUS BC
SUMMER REVIEW
MR. GOAR

DIRECTIONS:

- 1. THIS IS DUE FRIDAY, AUGUST 19TH, 2011.**
- 2. FOR ALL WORK, WRITE CLEARLY AND LEGIBLY.**
- 3. COPY EACH PROBLEM GIVEN ONTO YOUR OWN PAPER.**
- 4. SHOW EACH AND EVERY STEP IN OBTAINING YOUR SOLUTION.**
- 5. CIRCLE YOUR ANSWER(S) AND IF ASKED, CLARIFY YOUR REASONING.**
- 6. WRITE ALL ANSWERS, INCLUDING YOUR GRAPHS, ON THE ANSWER SHEET.**

Limits

1. Find $\lim_{x \rightarrow 0} \frac{\sqrt{x+3} - \sqrt{3}}{x}$.

2. Find $\lim_{x \rightarrow 2} \frac{1}{x-2} - \frac{4}{x^2-4}$.

3. Find $\lim_{x \rightarrow +\infty} \frac{4x-1}{\sqrt{x^2+2}}$.

4. Evaluate $\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2+5}}{3x^2-2}$.

5. Find the vertical and horizontal asymptotes of the graph of the function $f(x) = \frac{(2x+3)}{\sqrt{x^2-2x-3}}$.

6. Find the vertical and horizontal asymptotes of the graph of the function $f(x) = \sqrt{x+1} - \sqrt{x}$.

7. Find the vertical and horizontal asymptotes of the graph of the function $f(x) = \frac{(x^2-5x+6)}{(x-3)}$.

8. Evaluate $\lim_{x \rightarrow \infty} \frac{\sqrt{x+3} - \sqrt{3}}{x}$.

9. Evaluate $\lim_{x \rightarrow 2} \frac{\sqrt{x^2+5} - 3}{x^2-2x}$.

10. Find $\lim_{x \rightarrow -\infty} \frac{4x-1}{\sqrt{x^2+2}}$.

Continuity

11. Find the points of discontinuity (if any) of the function $f(x)$ such that $f(x) = \begin{cases} x+1 & \text{if } x \geq 2 \\ 2x-1 & \text{if } 1 < x < 2 \\ x-1 & \text{if } x \leq 1 \end{cases}$.

12. Find the points of discontinuity (if any) of $f(x) = \frac{3x+3}{x^2-3x-4}$, and write an equation for each vertical and horizontal asymptote of the graph of f .

13. If the function $f(x) = \begin{cases} \frac{x^2-16}{x-4} & \text{if } x \neq 4 \\ C & \text{if } x = 4 \end{cases}$ is continuous, what is the value of C ?

14. For what value of k is the following a continuous function?

$$f(x) = \begin{cases} \frac{\sqrt{7x+2} - \sqrt{6x+4}}{x-2} & \text{if } x \geq -\frac{2}{7} \text{ and } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$$

15. Show that the function $f(x) = 2x^3 - 4x^2 + 5x - 4$ has a zero between $x = 1$ and $x = 2$.

16. Let $f(x) = \begin{cases} 3x^2 - 1 & \text{if } x < 0 \\ cx + d & \text{if } 0 \leq x \leq 1. \\ \sqrt{x+8} & \text{if } x > 1 \end{cases}$. Determine c and d so that f is continuous everywhere.

Derivatives

17. Find the derivative of $G(x) = \frac{3x-2}{x^2+7}$.

18. Find the derivative of $H(x) = \frac{3x-2}{2x+5}$.

19. Find the slope-int. equation of the tangent line to the graph of $f(x) = 4x^3 - 7x^2$ at the point where $x = 3$.

20. Specify all lines through the point $(1, 5)$ and tangent to the curve $y = 3x^3 + x + 4$.

21. Find the slope-int. equation of the normal line to the graph of $f(x) = x^3 - x^2$ at the point where $x = 1$.

22. If the line $4x - 9y = 0$ is tangent in the first quadrant to the graph of $y = \frac{1}{3}x^3 + c$, what is the value of c ?

23. Find the points on the curve $y = \frac{1}{3}x^3 - x$ where the tangent line is parallel to the line $y = 3x$.

24. Find the point(s) at which the tangent line to the parabola $y = ax^2 + bx + c$ is horizontal.

25. Where does the normal line to the curve $y = x - x^2$ at the point $(1, 0)$ intersect the curve a second time?

26. Find the point(s) on the graph of $y = x^2$ at which the tangent line is parallel to the line $y = 6x - 1$.

27. Find the point(s) on the graph $y = x^3$ at which the tangent line is perpendicular to the line $3x + 9y = 4$.

28. Find the point(s) on the graph of $y = x^2$ at which the tangent line passes through $(2, -12)$.

29. Let $f(x) = 3x^3 - 11x^2 - 15x + 63$. Find all points on the graph of f where the tangent line is horizontal.

30. Find the derivative of $\frac{3x^7 + x^5 - 2x^4 + x - 3}{x^4}$.

The Chain Rule

31. Find the derivative of $\frac{1}{(3x^2 + 5)^4}$.

32. Find the derivative of $\left(\frac{x+2}{x-3}\right)^3$.

33. Find the derivative of $(4x^2 - 3)^2(x + 5)^3$.

34. Find the derivative of $\sqrt{\frac{4}{x}} - \sqrt{3x}$.

35. Find the derivative of $\sqrt{4 - \sqrt{4 + x}}$.

36. Find the derivative of $\sqrt[3]{(1 + x^2)^4}$.

37. Find the slope-int. equation of the tangent line to the graph of $y = \frac{\sqrt{x-1}}{x^2+1}$ at the point $(2, \frac{1}{5})$.
38. If $g(x) = x^{1/5}(x-1)^{3/5}$, find the domain of $g'(x)$.
39. Let F and G be differentiable functions such that $F(3) = 5$, $G(3) = 7$, $F'(3) = 13$, $G'(3) = 6$, $F'(7) = 2$, and $G'(7) = 0$. If $H(x) = F(G(x))$, find $H'(3)$.
40. Let $F(x) = \sqrt{1+3x}$. Find the coordinates of the point(s) on the graph of F where the normal line is parallel to the line $4x + 3y = 1$.
41. A point moves along the curve $y = x^3 - 3x + 5$ so that $x = \frac{1}{2}\sqrt{t} + 3$, where t is time. At what rate is y changing when $t = 4$?
42. Find the derivative of $(3x^2 + 5)^{-3}$.
43. Find the derivative of $\left(\frac{x^2-2}{2x^2+1}\right)^2$.
44. Find the derivative of $x^2(1-3x^3)^{1/3}$.
45. Find the derivative of $\sqrt[3]{(4x^2+3)^2}$.

Trigonometric Functions and Their Derivatives

46. Calculate $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x}$.
47. Find $\frac{d}{dx} \sqrt{\sin x}$.
48. Find $D_x(\csc \sqrt{x})$.
49. Find an equation of the tangent line to the graph of $y = \sin^2 x$ at the point where $x = \pi/3$.
50. Find an equation of the tangent line to the curve $y = \tan^2 x$ at the point $(\pi/3, 3)$.
51. Evaluate $\lim_{x \rightarrow 0} \frac{\tan^3 2x}{x^3}$.
52. Find $D_x(x^2 \cos 2x)$.
53. Find $\frac{d}{dx} \sin^3(5x+4)$.
54. Find $\frac{d}{dx} 2 \tan\left(\frac{x}{2}\right) - 5$.
55. Evaluate $\lim_{x \rightarrow +\infty} \frac{4x^3 - 1}{3x^2 + 2} \sin\left(\frac{1}{x}\right)$.

Rolle's Theorem, the Mean Value Theorem, and the Sign of the Derivative

Determine whether the hypotheses of Rolle's Theorem hold for the function $f(x)$ on the given interval, and if they do, verify the conclusion of the theorem.

56. $f(x) = x^3 - x$ on $[0, 1]$
57. $f(x) = \frac{x^2 - x - 6}{x - 1}$ on $[-2, 3]$
58. $f(x) = x^{2/3} - 2x^{1/3}$ on $[0, 8]$

Determine whether the hypotheses of the Mean Value Theorem hold for the function $f(x)$ on the given interval, and if they do, find a value c satisfying the conclusion of the theorem.

59. $f(x) = 3x^2 - 5x + 1$ on $[2, 5]$
60. $f(x) = x^{3/4}$ on $[0, 16]$
61. $f(x) = \frac{x+3}{x-4}$ on $[1, 3]$

62. $f(x) = \sqrt{25 - x^2}$ on $[-3, 4]$

Determine where the function $f(x)$ is increasing and where it is decreasing.

63. $f(x) = \sqrt{1 - x^2}$

64. $f(x) = x^3 - 9x^2 + 15x - 3$

65. $f(x) = x + \frac{1}{x}$

Higher Order Derivatives and Implicit Differentiation

66. Find y'' of $y = \sqrt{x^2 + 1}$.

67. Find the smallest positive integer n such that $D_x^n(\cos x) = \cos x$.

68. Calculate $y^{(5)}$ for $y = \sin^2 x$.

69. On the circle $x^2 + y^2 = a^2$, find y'' .

70. If $x^3 - y^3 = 1$, find y'' .

71. If $xy + y^2 = 1$, find y' and y'' .

72. At the point $(1, 2)$ of the curve $x^2 - xy + y^2 = 3$, find an equation of the tangent line.

73. If $x^2 + 2xy + 3y^2 = 2$, find y' and y'' when $y = 1$.

74. Find an equation of the tangent line to the curve $1 + 16x^2y = \tan(x - 2y)$ at the point $(\pi/4, 0)$.

75. Find the 10th and 11th derivatives of $f(x) = x^{10} - 14x^7 + 3x^5 + 2x^3 - x + 2$.

76. Find y'' of $x^2 - y^2 = 1$.

77. Find y' of $\tan xy = y$.

78. Find y' of $\tan^2(y + 1) = 3 \sin x$.

79. Find the slope of the tangent line to the curve $x^2 + 2xy - 3y^2 = 9$ at the point $(3, 2)$.

80. Show that the circles $x^2 + y^2 - 12x - 6y + 25 = 0$ and $x^2 + y^2 + 2x + y - 10 = 0$ are tangent to each other at the point $(2, 1)$.

Maxima and Minima

81. Find the critical numbers of $f(x) = \frac{x^2}{x-1}$, and determine whether they yield relative maxima, relative minima, or inflection points.

82. Find the critical numbers of $f(x) = (x-1)^{2/3}$, and determine whether they yield relative maxima, relative minima, or inflection points.

83. Find the critical numbers of $f(x) = \sin x - x$, and determine whether they yield relative maxima, relative minima, or inflection points.

84. Find the absolute maximum and minimum of the function $f(x) = 4x^2 - 7x + 3$ on $[-2, 3]$.

85. Find the absolute maximum and minimum of the function $f(x) = \frac{x^3}{x+2}$ on $[-1, 1]$.

86. Find the absolute extrema of the function $f(x) = \sin x - \cos x$ on $[0, \pi]$.
87. Find the points at which the function $f(x) = (x - 2)^4(x + 1)^3$ has relative extrema.
88. Find the absolute maximum and minimum of the function $f(x) = \frac{x^2}{16} + \frac{1}{x}$ on $[1, 4]$.
89. Find the absolute maximum and minimum of the function $f(x) = 2 \sin x + \sin 2x$ on $[0, 2\pi]$.
90. Find the absolute maximum and minimum of the function $f(x) = \left| \cos x - \frac{1}{2} \right|$ on $[0, 2\pi]$.

Related Rates

91. The top of a 25-foot ladder, leaning against a vertical wall is slipping down the wall at the rate of 1 foot per second. How fast is the bottom of the ladder slipping along the ground when the bottom of the ladder is 7 feet away from the base of the wall?
92. A rocket is shot vertically upward with an initial velocity of 400 feet per second. Its height s after t seconds is $s = 400t - 16t^2$. How fast is the distance changing from the rocket to the observer on the ground 1800 feet away from the launching site, when the rocket is still rising and is 2400 feet above the ground?
93. A small funnel in the shape of a cone is being emptied of fluid at the rate of 12 cubic centimeters per second. The height of the funnel is 20 centimeters and the radius of the top is 4 centimeters. How fast is the fluid level dropping when the level stands 5 centimeters above the vertex of the cone? ($V_{Cone} = \frac{1}{3}\pi r^2 h$)
94. A balloon is being inflated by pumped air at the rate of 2 cubic inches per second. How fast is the diameter of the balloon increasing when the radius is $\frac{1}{2}$ inch?
95. Oil from an uncapped well in the ocean is radiating outward in the form of a circular film on the surface of the water. If the radius of the circle is increasing at the rate of 2 meters per minute, how fast is the area of the oil film growing when the radius is 100 meters?
96. A particle moves on the hyperbola $x^2 - 18y^2 = 9$ in such a way that its y -coordinate increases at a constant rate of 9 units per second. How fast is its x -coordinate changing when $x = 9$?
97. A boat passes a fixed buoy at 9 a.m. heading due west at 3 miles per hour. Another boat passes the same buoy at 10 a.m. heading due north at 5 miles per hour. How fast is the distance between the boats changing at 11:30 a.m.?
98. A particle moves along the curve $y = x^2 + 2x$. At what point(s) on the curve are the x - and y -coordinates of the particle changing at the same rate?
99. A boat is being pulled into a dock by a rope that passes through a ring on the bow of the boat. The dock is 8 feet higher than the bow ring. How fast is the boat approaching the dock when the length of rope between the dock and the boat is 10 feet, if the rope is being pulled in at the rate of 3 feet per second?
100. A spherical snowball is melting (symmetrically) at the rate of 4π cubic centimeters per hour. How fast is the diameter changing when it is 20 centimeters?

Curve Sketching (Graphs)

Sketch the graph of the given function, showing all relative extrema, inflection points, and asymptotes; indicate concavity; and suggest the behavior at infinity.

101. $f(x) = (x^2 - 1)^3$

102. $f(x) = x^3 - 2x^2 - 4x + 3$

103. $f(x) = x^2 + \frac{2}{x}$

104. $f(x) = \frac{x^2 - 3}{x^3}$

105. $f(x) = \sin x + \cos x$

106. $f(x) = \frac{2x^2}{x^2 + 3}$

107. $f(x) = \frac{3}{5}x^{5/3} - 3x^{2/3}$

Rectilinear Motion

108. The position of a moving object on a line is given by the formula $s = (t - 1)^3(t - 5)$. When is the object moving to the right, when is it moving left, when does it change direction, and when is it at rest? What is the farthest to the left of the origin that it moves?

109. A particle moves along the x -axis according to the equation $x = 10t - 2t^2$. What is the total distance covered by the particle between $t = 0$ and $t = 3$?

110. Two particles move along the x -axis. Their positions $f(t)$ and $g(t)$ are given by $f(t) = 6t - t^2$ and $g(t) = t^2 - 4t$. (a) When do they have the same position? (b) When do they have the same velocity? (c) When they have the same position, are they moving in opposite directions?

The Natural Logarithm

111. Find $\frac{d}{dx}(\ln x)^3$.

112. Find $D_x \ln\left(\frac{x-1}{x+1}\right)$.

Use logarithmic differentiation to find y' .

113. $y = x^3\sqrt{4-x^2}$

114. $y = \frac{\sqrt{x^2-1} \cdot \sin x}{(2x+3)^4}$

115. Graph $\ln(\cos x)$.

116. Find y' when $y^2 = \ln(x^2 + y^2)$.

117. Evaluate $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x}$.

118. Sketch the graph of $y = 3x + 1 - 5\ln(1+x^2)$.

Exponential Functions

119. Find $\frac{d}{dx}e^{1/x}$.

120. Find $D_x e^{\cos x}$.

121. Find $\frac{d}{dx} \frac{e^x}{x}$.

122. Find $\frac{d}{dx} x^\pi$.

123. Find $D_x \pi^x$.

124. Find y' for $e^y = y + \ln x$.

125. Find y' for $\tan e^{y-x} = x^2$.

126. Find y' for $x^2 + e^{xy} + y^2 = 1$.

127. Find y' for $\sin x = e^y$.

128. Find y' for $y = 3^{\sin x}$.

129. Find y' for $y = x^{\ln x}$.

130. If $y = 2e^{\sin x}$, find y' and y'' .

131. Find an equation of the tangent line to the curve $y = 2e^x$ at the point $(0, 2)$.

132. Graph $y = (1 - \ln x)^2$.

133. Graph $y = x^2 e^x$.

L'Hospital's Rule

134. Evaluate $\lim_{x \rightarrow +\infty} \frac{\ln(1 + e^x)}{1 + x}$.

135. Evaluate $\lim_{x \rightarrow 0} \frac{1 - e^x}{x}$.

136. Evaluate $\lim_{x \rightarrow 1} \frac{x^3 - x^2 + x - 1}{x + \ln x - 1}$.

137. Evaluate $\lim_{x \rightarrow 0} \left[\frac{1}{\ln(x+1)} - \frac{1}{x} \right]$.

138. Evaluate $\lim_{x \rightarrow +\infty} x^{1/x}$.

139. Evaluate $\lim_{x \rightarrow 0} \frac{3^x - 2^x}{x}$.

140. Evaluate $\lim_{x \rightarrow 0^+} x^{\sin x}$.

141. Evaluate $\lim_{x \rightarrow 1} \frac{\ln x}{\tan \pi x}$.

142. Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos^2 2x}{x}$.

143. Evaluate $\lim_{x \rightarrow +\infty} \frac{2^x}{x^{10}}$.

144. Evaluate $\lim_{x \rightarrow +\infty} \left(\cos \frac{1}{x} \right)^x$.

145. Evaluate $\lim_{x \rightarrow 0^+} \sin x^{\tan x}$.

Inverse Trigonometric Functions

146. Find $\frac{d}{dx} (\sin^{-1} x + \cos^{-1} x)$.

147. Find $\frac{d}{dx} (x \tan^{-1} x)$.

148. Find $D_x (\sin^{-1} \sqrt{x})$.

149. Find $D_x \ln(\cot^{-1} 3x)$.

150. Find the derivative of $y = \tan^{-1}(e^{\sin x})$.

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NAME: _____

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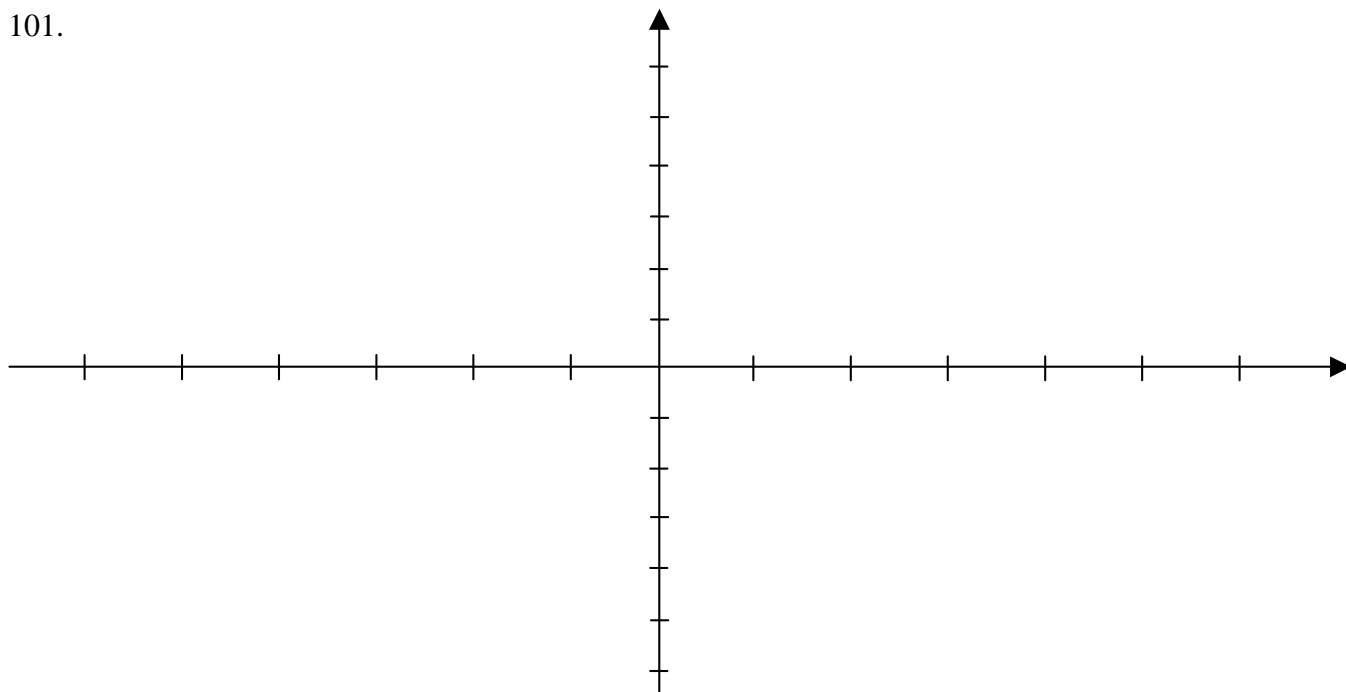
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61. _____	81. _____	101. ----(See Attached Graph)----
62. _____	82. _____	102. ----(See Attached Graph)----
63. _____	83. _____	103. ----(See Attached Graph)----
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76. _____	96. _____	116. _____
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78. _____	98. _____	118. ----(See Attached Graph)----
79. _____	99. _____	119. _____
80. _____	100. _____	120. _____

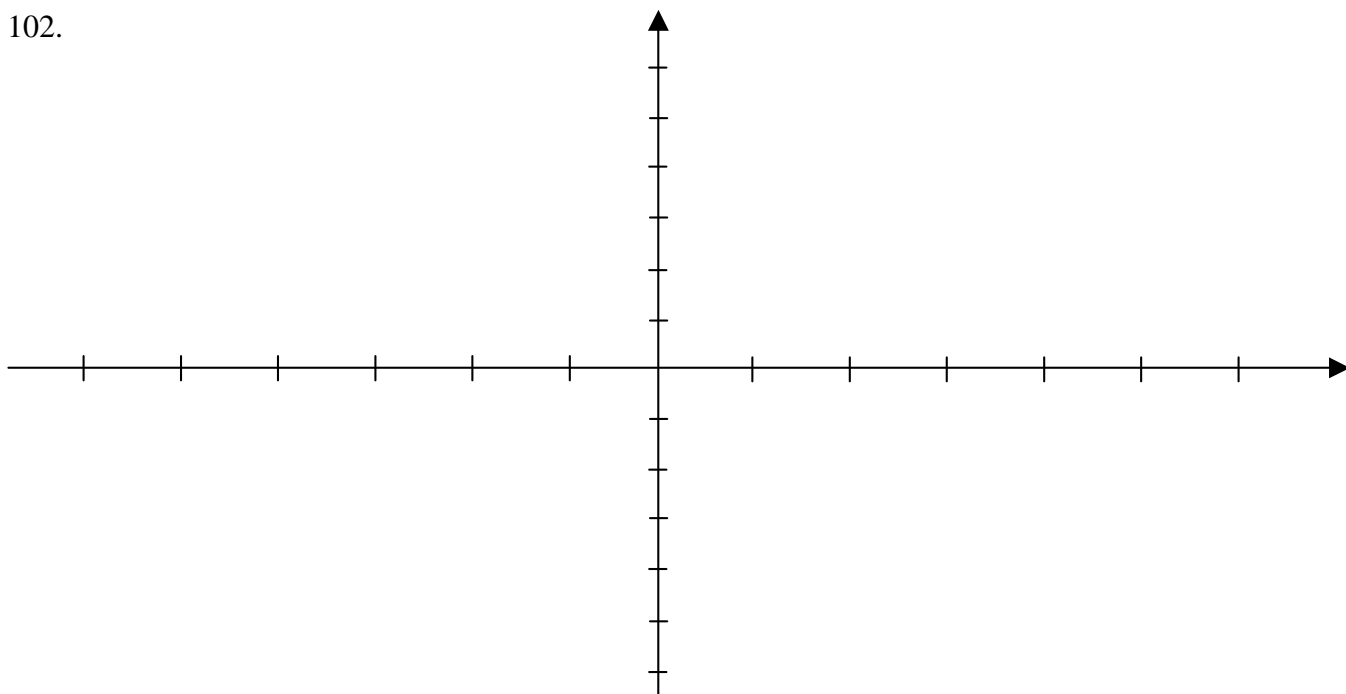
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GRAPHS:

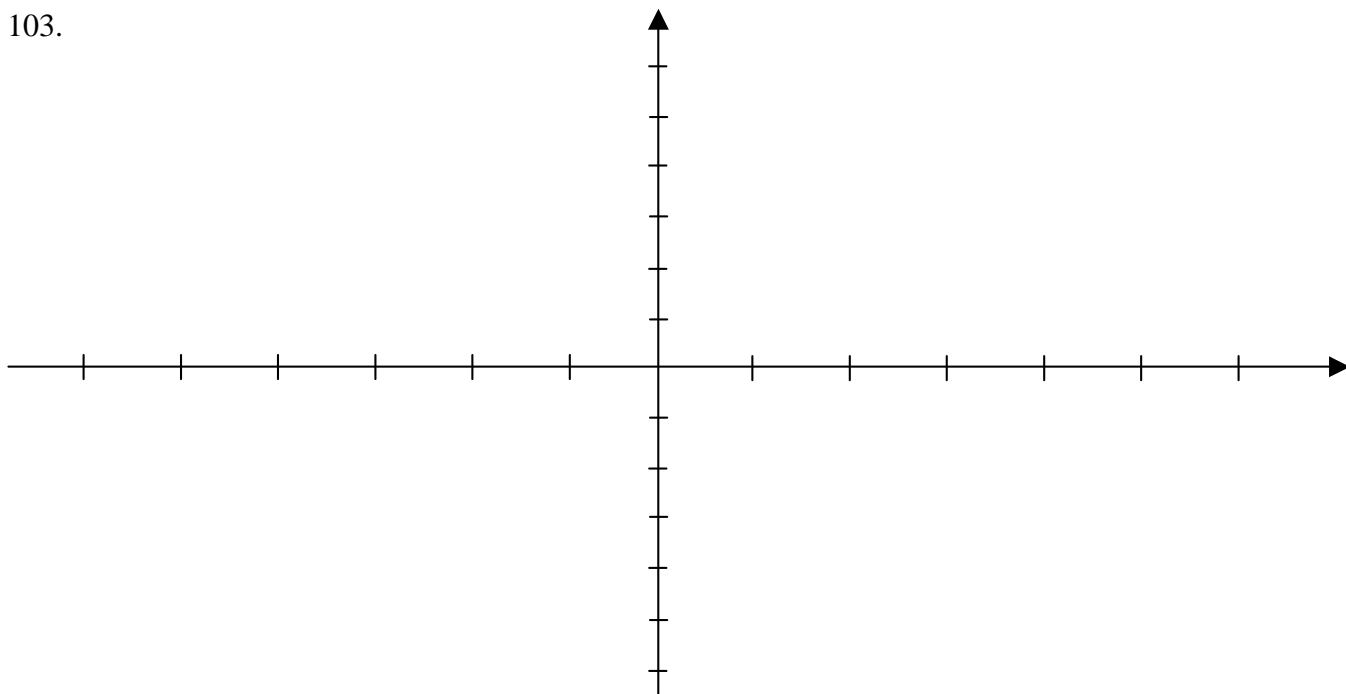
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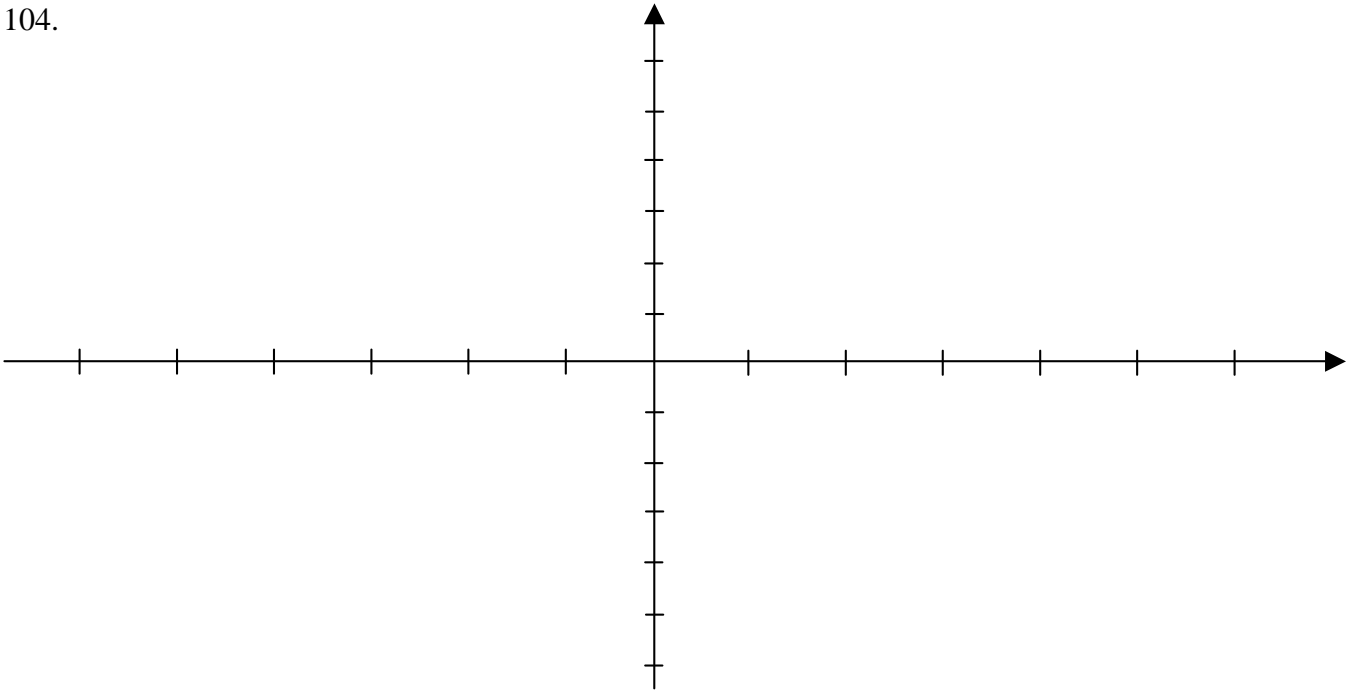
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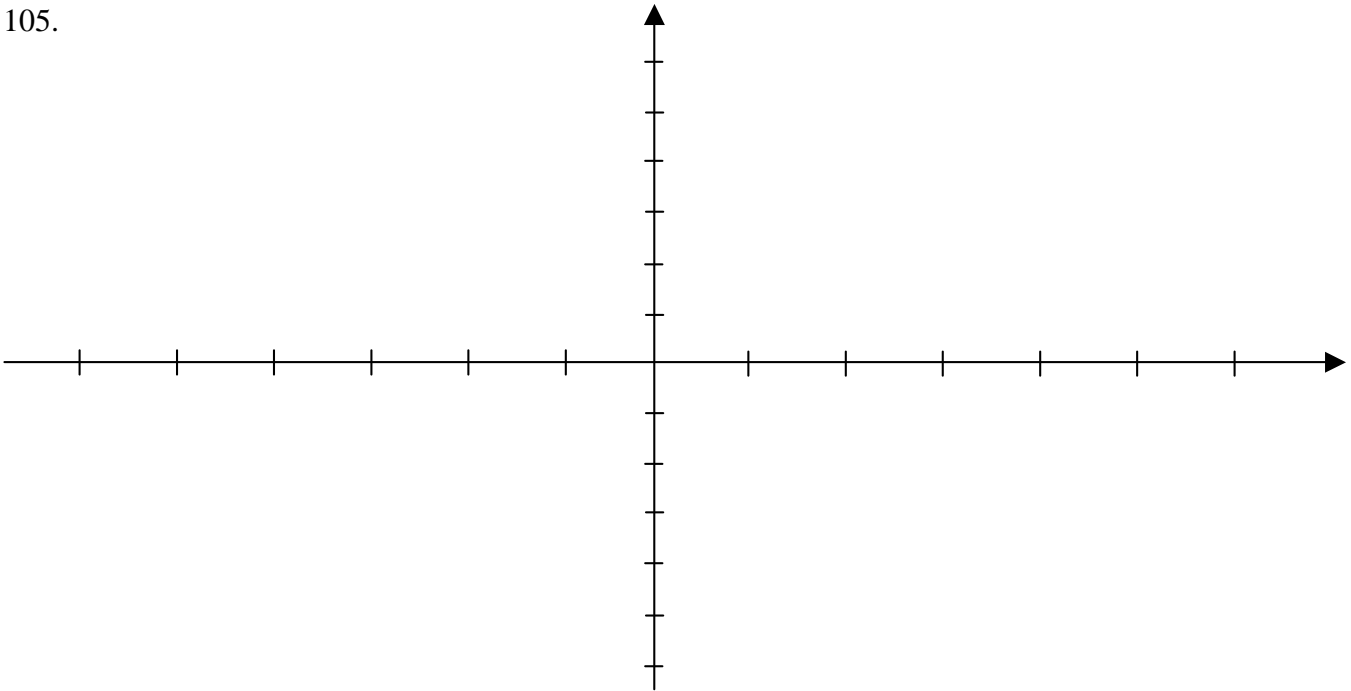
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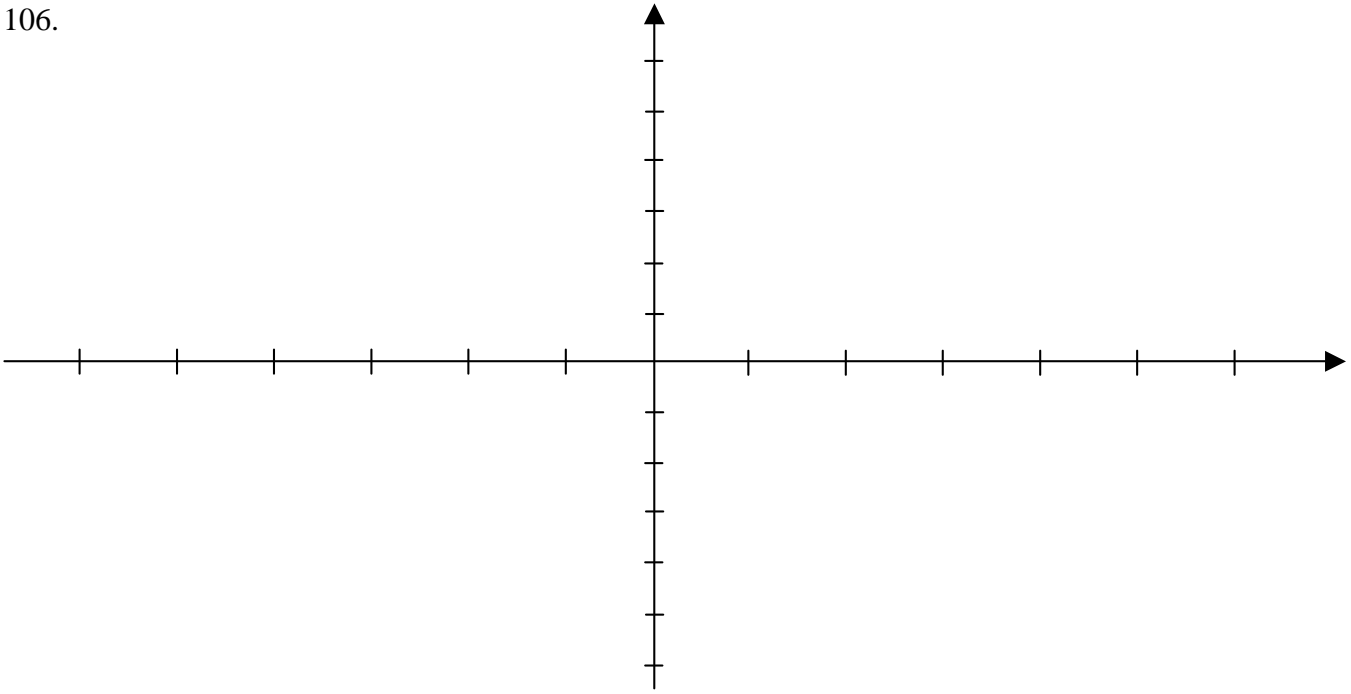
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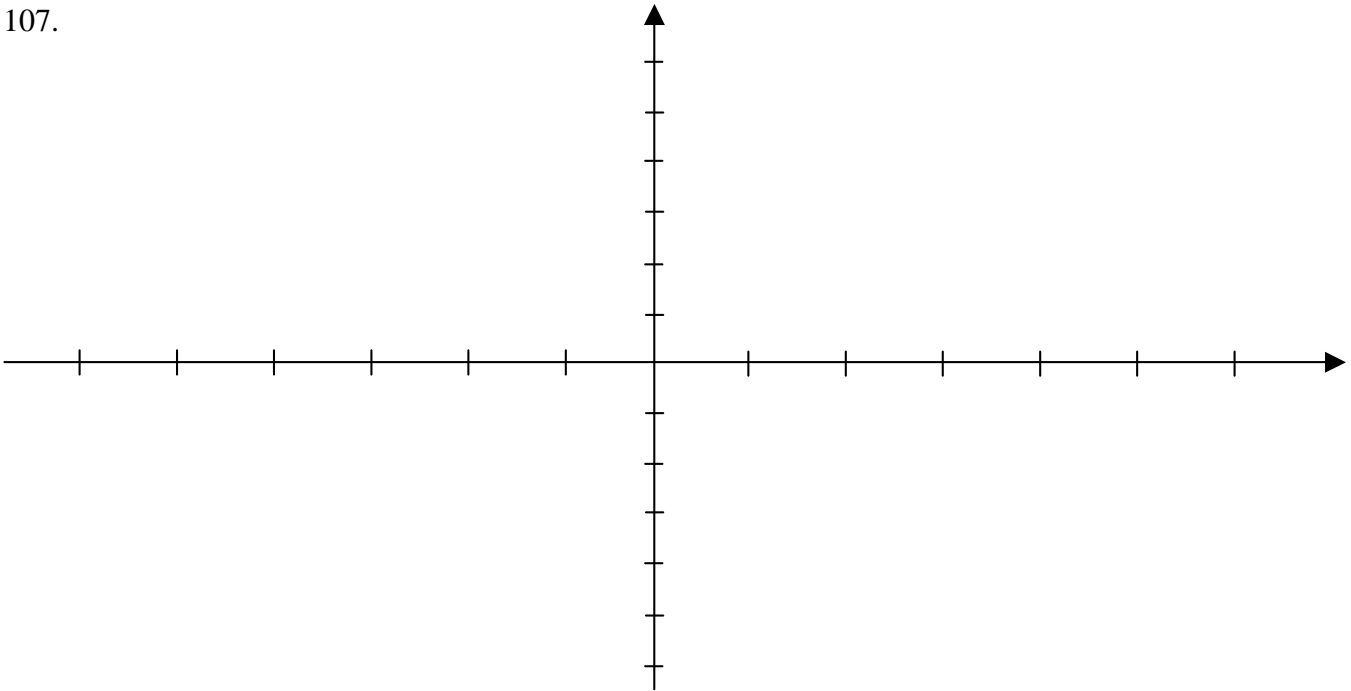
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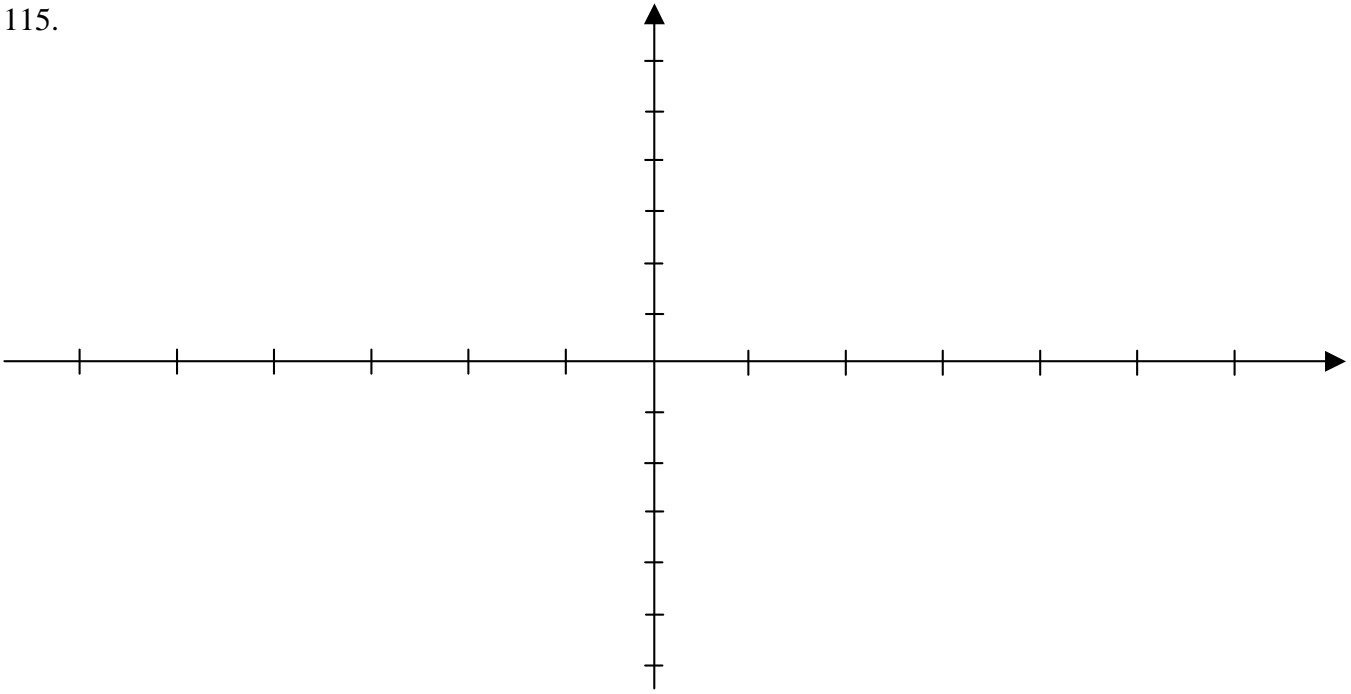
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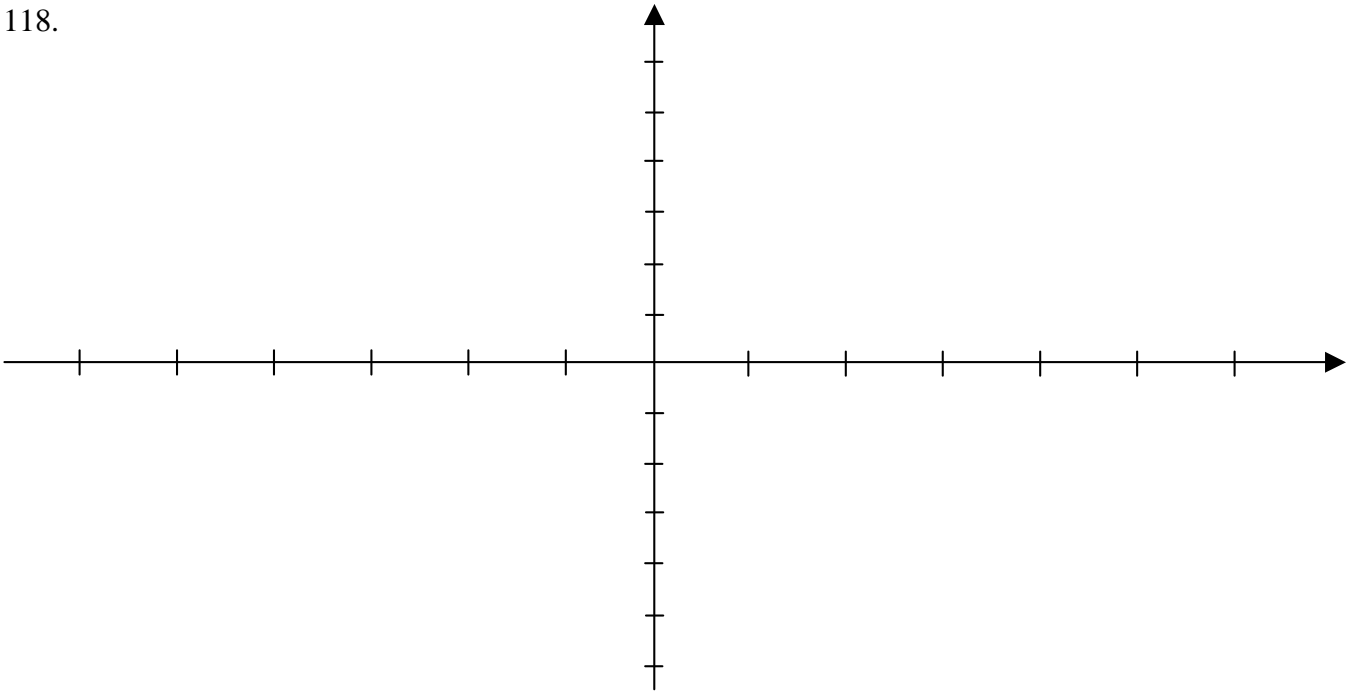
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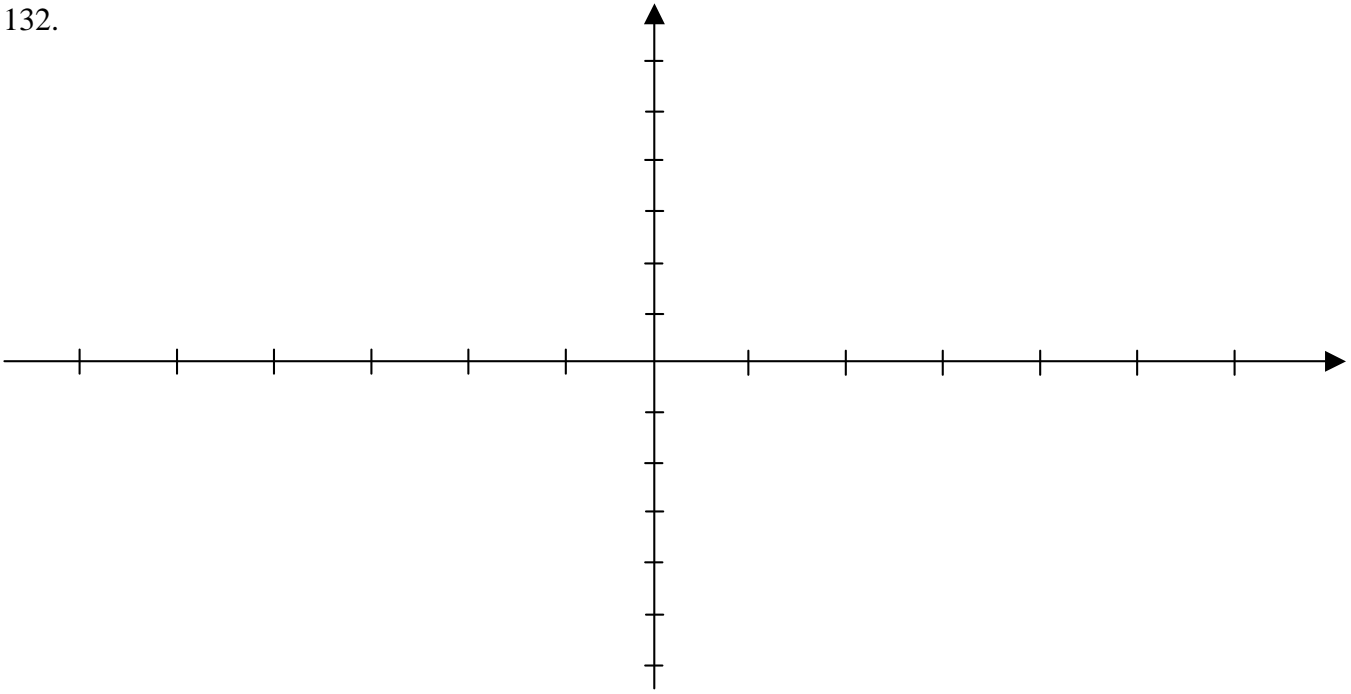
115.



118.



132.



133.

