Step 4 Check the proposed solutions in the original equation.

Check 1:

$$\sqrt{2x-1}+2=x$$
$$\sqrt{2\cdot 1-1}+2 \stackrel{?}{=} 1$$
$$\sqrt{1}+2 \stackrel{?}{=} 1$$
$$1+2 \stackrel{?}{=} 1$$
$$3=1, \quad \text{false}$$

Check 5:

$$\sqrt{2x-1}+2=x$$
$$\sqrt{2\cdot 5-1}+2 \stackrel{?}{=} 5$$
$$\sqrt{9}+2 \stackrel{?}{=} 5$$
$$3+2 \stackrel{?}{=} 5$$
$$5=5, \quad \text{true}$$

Thus, 1 is an extraneous solution. The only solution is 5, and the solution set is $\{5\}$.

Check Point 12 Solve: $\sqrt{x+3}+3=x$.

EXERCISE SET P.7

Practice Exercises

In Exercises 1–16, solve each linear equation.

1. $7x-5=72$
2. $6x-3=63$
3. $11x-(6x-5)=40$
4. $5x-(2x-10)=35$
5. $2x-7=6+x$
6. $3x+5=2x+13$
7. $7x+4=x+16$
8. $13x+14=12x-5$
9. $3(x-2)+7=2(x+5)$
10. $2(x-1)+3=x-3(x+1)$
11. $\frac{x+3}{6}=\frac{3}{8}+\frac{x-5}{4}$
12. $\frac{x+1}{4}=\frac{1}{6}+\frac{2-x}{3}$
13. $\frac{x}{4}=2+\frac{x-3}{3}$
14. $5+\frac{x-2}{3}=\frac{x+3}{8}$
15. $\frac{x+1}{3}=5-\frac{x+2}{7}$
16. $\frac{3x}{5}-\frac{x-3}{2}=\frac{x+2}{3}$

Exercises 17–26 contain rational equations with variables in denominators. For each equation, **a.** *Write the value or values of the variable that make a denominator zero. These are the restrictions on the variable.* **b.** *Keeping the restrictions in mind, solve the equation.*

17. $\frac{1}{x-1}+5=\frac{11}{x-1}$
18. $\frac{3}{x+4}-7=\frac{-4}{x+4}$
19. $\frac{8x}{x+1}=4-\frac{8}{x+1}$
20. $\frac{2}{x-2}=\frac{x}{x-2}-2$
21. $\frac{3}{2x-2}+\frac{1}{2}=\frac{2}{x-1}$
22. $\frac{3}{x+3}=\frac{5}{2x+6}+\frac{1}{x-2}$
23. $\frac{2}{x+1}-\frac{1}{x-1}=\frac{2x}{x^2-1}$
24. $\frac{4}{x+5}+\frac{2}{x-5}=\frac{32}{x^2-25}$
25. $\frac{1}{x-4}-\frac{5}{x+2}=\frac{6}{x^2-2x-8}$
26. $\frac{1}{x-3}-\frac{2}{x+1}=\frac{8}{x^2-2x-3}$

In Exercises 27–42, solve each formula for the specified variable. Do you recognize the formula? If so, what does it describe?

27. $I=Prt$ for P
28. $C=2\pi r$ for r
29. $T=D+pm$ for p
30. $P=C+MC$ for M
31. $A=\frac{1}{2}h(a+b)$ for a
32. $A=\frac{1}{2}h(a+b)$ for b
33. $S=P+Prt$ for r
34. $S=P+Prt$ for t
35. $B=\frac{F}{S-V}$ for S
36. $S=\frac{C}{1-r}$ for r
37. $IR+Ir=E$ for I
38. $A=2lw+2lh+2wh$ for h
39. $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ for f
40. $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$ for R_1
41. $f=\frac{f_1f_2}{f_1+f_2}$ for f_1
42. $f=\frac{f_1f_2}{f_1+f_2}$ for f_2

In Exercises 43–54, solve each absolute value equation or indicate the equation has no solution.

43. $|x-2|=7$
44. $|x+1|=5$
45. $|2x-1|=5$
46. $|2x-3|=11$
47. $2|3x-2|=14$
48. $3|2x-1|=21$
49. $2\left|4-\frac{5}{2}x\right|+6=18$
50. $4\left|1-\frac{3}{4}x\right|+7=10$
51. $|x+1|+5=3$
52. $|x+1|+6=2$
53. $|2x-1|+3=3$
54. $|3x-2|+4=4$

In Exercises 55–60, solve each quadratic equation by factoring.

55. $x^2-3x-10=0$
56. $x^2-13x+36=0$
57. $x^2=8x-15$
58. $x^2=-11x-10$
59. $5x^2=20x$
60. $3x^2=12x$

In Exercises 61–66, solve each quadratic equation by the square root property.

61. $3x^2 = 27$ **62.** $5x^2 = 45$

63. $5x^2 + 1 = 51$ **64.** $3x^2 - 1 = 47$

65. $3(x - 4)^2 = 15$ **66.** $3(x + 4)^2 = 21$

In Exercises 67–74, solve each quadratic equation by completing the square.

67. $x^2 + 6x = 7$ **68.** $x^2 + 6x = -8$

69. $x^2 - 2x = 2$ **70.** $x^2 + 4x = 12$

71. $x^2 - 6x - 11 = 0$ **72.** $x^2 - 2x - 5 = 0$

73. $x^2 + 4x + 1 = 0$ **74.** $x^2 + 6x - 5 = 0$

In Exercises 75–82, solve each quadratic equation using the quadratic formula.

75. $x^2 + 8x + 15 = 0$ **76.** $x^2 + 8x + 12 = 0$

77. $x^2 + 5x + 3 = 0$ **78.** $x^2 + 5x + 2 = 0$

79. $3x^2 - 3x - 4 = 0$ **80.** $5x^2 + x - 2 = 0$

81. $4x^2 = 2x + 7$ **82.** $3x^2 = 6x - 1$

Compute the discriminant of each equation in Exercises 83–90. What does the discriminant indicate about the number and type of solutions?

83. $x^2 - 4x - 5 = 0$ **84.** $4x^2 - 2x + 3 = 0$

85. $2x^2 - 11x + 3 = 0$ **86.** $2x^2 + 11x - 6 = 0$

87. $x^2 = 2x - 1$ **88.** $3x^2 = 2x - 1$

89. $x^2 - 3x - 7 = 0$ **90.** $3x^2 + 4x - 2 = 0$

In Exercises 91–114, solve each quadratic equation by the method of your choice.

91. $2x^2 - x = 1$ **92.** $3x^2 - 4x = 4$

93. $5x^2 + 2 = 11x$ **94.** $5x^2 = 6 - 13x$

95. $3x^2 = 60$ **96.** $2x^2 = 250$

97. $x^2 - 2x = 1$ **98.** $2x^2 + 3x = 1$

99. $(2x + 3)(x + 4) = 1$ **100.** $(2x - 5)(x + 1) = 2$

101. $(3x - 4)^2 = 16$ **102.** $(2x + 7)^2 = 25$

103. $3x^2 - 12x + 12 = 0$ **104.** $9 - 6x + x^2 = 0$

105. $4x^2 - 16 = 0$ **106.** $3x^2 - 27 = 0$

107. $x^2 = 4x - 2$ **108.** $x^2 = 6x - 7$

109. $2x^2 - 7x = 0$ **110.** $2x^2 + 5x = 3$

111. $\frac{1}{x} + \frac{1}{x+2} = \frac{1}{3}$ **112.** $\frac{1}{x} + \frac{1}{x+3} = \frac{1}{4}$

113. $\frac{2x}{x-3} + \frac{6}{x+3} = -\frac{28}{x^2-9}$

114. $\frac{3}{x-3} + \frac{5}{x-4} = \frac{x^2-20}{x^2-7x+12}$

In Exercises 115–124, solve each radical equation. Check all proposed solutions.

115. $\sqrt{3x + 18} = x$ **116.** $\sqrt{20 - 8x} = x$

117. $\sqrt{x + 3} = x - 3$ **118.** $\sqrt{x + 10} = x - 2$

119. $\sqrt{2x + 13} = x + 7$ **120.** $\sqrt{6x + 1} = x - 1$

121. $x - \sqrt{2x + 5} = 5$ **122.** $x - \sqrt{x + 11} = 1$

123. $\sqrt{2x + 19} - 8 = x$ **124.** $\sqrt{2x + 15} - 6 = x$

Practice Plus

In Exercises 125–134, solve each equation.

125. $25 - [2 + 5x - 3(x + 2)] = -3(2x - 5) - [5(x - 1) - 3x + 3]$

126. $45 - [4 - 2x - 4(x + 7)] = -4(1 + 3x) - [4 - 3(x + 2) - 2(2x - 5)]$

127. $7 - 7x = (3x + 2)(x - 1)$

128. $10x - 1 = (2x + 1)^2$

129. $|x^2 + 2x - 36| = 12$

130. $|x^2 + 6x + 1| = 8$

131. $\frac{1}{x^2 - 3x + 2} = \frac{1}{x + 2} + \frac{5}{x^2 - 4}$

132. $\frac{x - 1}{x - 2} + \frac{x}{x - 3} = \frac{1}{x^2 - 5x + 6}$

133. $\sqrt{x + 8} - \sqrt{x - 4} = 2$

134. $\sqrt{x + 5} - \sqrt{x - 3} = 2$

Exercises 135–136, list all number that must be excluded from the domain of each rational expression.

135. $\frac{3}{2x^2 + 4x - 9}$ **136.** $\frac{7}{2x^2 - 8x + 5}$

Application Exercises

In the section opener, we used two formulas to model the level of depression, D, in response to the intensity of a negative life event, x, from 1, low, to 10, high:

Low-Humor Group	High-Humor Group
$D = \frac{10}{9}x + \frac{53}{9}$	$D = \frac{1}{9}x + \frac{26}{9}$.

Use these formulas to solve Exercises 137–138.

137. If the high-humor group averages a level of depression of 3.5, or $\frac{7}{2}$, in response to a negative life event, what is the intensity of that event? How is the solution shown on the line graph in Figure P.12 on page 77?

138. If the low-humor group averages a level of depression of 10 in response to a negative life event, what is the intensity of that event? How is the solution shown on the line graph in Figure P.12 on page 77?

139. A company wants to increase the 10% peroxide content of its product by adding pure peroxide (100% peroxide). If x liters of pure peroxide are added to 500 liters of its 10% solution, the concentration, C, of the new mixture is given by

$$C = \frac{x + 0.1(500)}{x + 500}.$$

How many liters of pure peroxide should be added to produce a new product that is 28% peroxide?

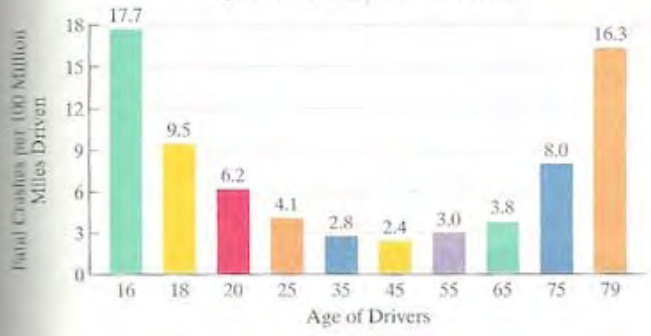
140. Suppose that x liters of pure acid are added to 200 liters of a 35% acid solution.

a. Write a formula that gives the concentration, C, of the new mixture. (*Hint*: See Exercise 139.)

b. How many liters of pure acid should be added to produce a new mixture that is 74% acid?

A driver's age has something to do with his or her chance of getting into a fatal car crash. The bar graph shows the number of fatal vehicle crashes per 100 million miles driven for drivers of various age groups. For example, 25-year-old drivers are involved in 4.1 fatal crashes per 100 million miles driven. Thus, when a group of 25-year-old Americans have driven a total of 100 million miles, approximately 4 have been in accidents in which someone died.

Age of U.S. Drivers and Fatal Crashes



Source: Insurance Institute for Highway Safety

The number of fatal vehicle crashes per 100 million miles, y, for drivers of age x can be modeled by the formula

$$y = 0.013x^2 - 1.19x + 28.24.$$

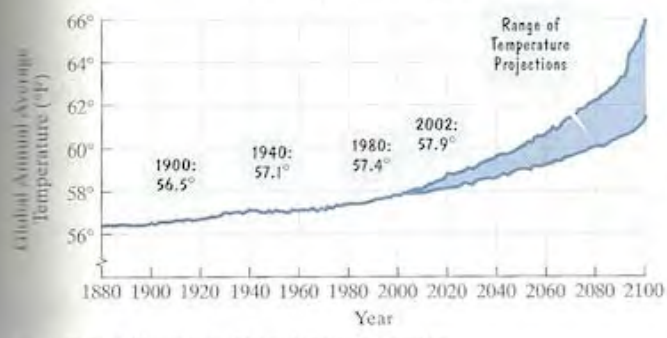
Use the formula to solve Exercises 141–142.

141. What age groups are expected to be involved in 3 fatal crashes per 100 million miles driven? How well does the formula model the trend in the actual data shown in the bar graph?

142. What age groups are expected to be involved in 10 fatal crashes per 100 million miles driven? How well does the formula model the trend in the actual data shown in the bar graph ?

In 2002, the average surface temperature on Earth was 57.9°F, approximately 1.4° higher than it was one hundred years ago. Worldwide temperatures have risen only 9°F since the end of the last ice age 12,000 years ago. Most climatologists are convinced that over the next one hundred years, global temperatures will continue to increase, possibly setting off a chain of devastating events beginning with a rise in sea levels worldwide and ending with the destruction of water supplies, forests, and agriculture in many parts of the world. The graph shows global annual average temperatures from 1880 through 2002, with projections from 2002 through 2100.

Global Annual Average Temperatures and Projections through 2100



Source: National Oceanic and Atmospheric Administration

The temperature projections shown in the graph can be modeled by two equations:

$$H = 0.083x + 57.9$$ Models temperatures at the high end of the range

$$L = 0.36\sqrt{x} + 57.9.$$ Models temperatures at the low end of the range

In these equations, H and L describe projected global annual average temperatures, in degrees Fahrenheit, x years after 2002, where $0 \le x \le 98$. Use the models to solve Exercises 143–146.

143. Use H and L to determine the temperatures at the high and low end of the range of projected global average temperatures for 2100. Round to the nearest tenth of a degree.

144. Use H and L to determine the temperatures at the high and low end of the range of projected global average temperatures for 2080. Round to the nearest tenth of a degree.

145. Use H and L to determine by which year the projected global average temperature will exceed the 2002 average of 57.9° by one degree. Round to the nearest year.

146. Use H and L to determine by which year the projected global average temperature will exceed the 2002 average of 57.9° by two degrees.

Writing in Mathematics

147. What is a linear equation in one variable? Give an example of this type of equation.

148. Explain how to determine the restrictions on the variable for the equation

$$\frac{3}{x+5} + \frac{4}{x-2} = \frac{7}{x^2+3x-6}.$$

149. What does it mean to solve a formula for a variable?

150. Explain how to solve an equation involving absolute value.

151. Why does the procedure that you explained in Exercise 150 not apply to the equation $|x - 2| = -3$? What is the solution set for this equation?

152. What is a quadratic equation?

153. Explain how to solve $x^2 + 6x + 8 = 0$ using factoring and the zero-product principle.

154. Explain how to solve $x^2 + 6x + 8 = 0$ by completing the square.

155. Explain how to solve $x^2 + 6x + 8 = 0$ using the quadratic formula.

156. How is the quadratic formula derived?

157. What is the discriminant and what information does it provide about a quadratic equation?

158. If you are given a quadratic equation, how do you determine which method to use to solve it?

159. In solving $\sqrt{2x - 1} + 2 = x$, why is it a good idea to isolate the radical term? What if we don't do this and simply square each side? Describe what happens.

160. What is an extraneous solution to a radical equation?

Critical Thinking Exercises

161. Which one of the following is true?

a. The equation $(2x - 3)^2 = 25$ is equivalent to $2x - 3 = 5$.

b. Every quadratic equation has two distinct numbers in its solution set.

c. The equation $3y - 1 = 11$ and $3y - 7 = 5$ are equivalent.

d. The equation $ax^2 + c = 0, a \neq 0$, cannot be solved by the quadratic formula.

162. Find b such that $\frac{7x + 4}{b} + 13 = x$ will have a solution set given by $\{-6\}$.

163. Write a quadratic equation in general form whose solution set is $\{-3, 5\}$.

165. Solve for C: $V = C - \frac{C - S}{L}N$.

165. Solve for t: $s = -16t^2 + v_0t$.

SECTION P.8 Modeling with Equations

Objective

1. Use equations to solve problems.

The human race is undeniably becoming a faster race. Since the beginning of the past century, track-and-field records have fallen in everything from sprints to miles to marathons. The performance arc is clearly rising, but no one knows how much higher it can climb. At some point, even the best-trained body simply has to up and quit. The question is, just where is that point, and is it possible for athletes, trainers, and genetic engineers to push it higher? In this section, you will learn a problem-solving strategy that uses linear equations to determine if anyone will ever run a 3-minute mile.

1 Use equations to solve problems.

Problem Solving with Linear Equations

We have seen that a model is a mathematical representation of a real-world situation. In this section, we will be solving problems that are presented in English. This means that we must obtain models by translating from the ordinary language of English into the language of algebraic equations. To translate, however, we must understand the English prose and be familiar with the forms of algebraic language. Here are some general steps we will follow in solving word problems:

Study Tip

When solving word problems, particularly problems involving geometric figures, drawing a picture of the situation is often helpful. Label x on your drawing and, where appropriate, label other parts of the drawing in terms of x.

Strategy for Solving Word Problems

Step 1 Read the problem carefully. Attempt to state the problem in your own words and state what the problem is looking for. Let x (or any variable) represent one of the quantities in the problem.

Step 2 If necessary, write expressions for any other unknown quantities in the problem in terms of x.

Step 3 Write an equation in x that models the verbal conditions of the problem.

Step 4 Solve the equation and answer the problem's question.

Step 5 Check the solution *in the original wording* of the problem, not in the equation obtained from the words.

$$50{,}000x + 50{,}000 - 2500x^2 - 2500x = 50{,}000x$$ Use the distributive property.

$$-2500x^2 + 47{,}500x + 50{,}000 = 50{,}000x$$ Combine like terms: $50{,}000x - 2500x = 47{,}500x$.

$$-2500x^2 - 2500x + 50{,}000 = 0$$ Write the quadratic equation in general form, subtracting $50{,}000x$ from both sides.

$$-2500(x^2 + x - 20) = 0$$ Factor out -2500.

$$-2500(x + 5)(x - 4) = 0$$ Factor completely.

$$x + 5 = 0 \quad \text{or} \quad x - 4 = 0$$ Set each variable factor equal to zero.

$$x = -5 \qquad x = 4$$ Solve the resulting equations.

Because x represents the number of people in the original group, x cannot be negative. Thus, there were four people in the original group.

Step 5 Check the proposed solution in the original wording of the problem.

$$\text{original cost per person} = \frac{\$50{,}000}{4} = \$12{,}500$$

$$\text{final cost per person} = \frac{\$50{,}000}{5} = \$10{,}000$$

We see that each person's share is reduced by \$12,500 − \$10,000, or \$2500, as specified by the problem's conditions.

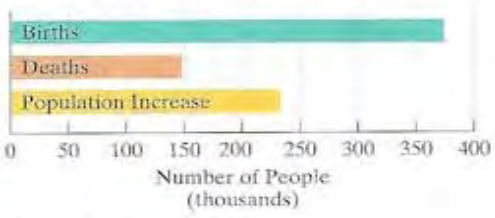
Check Point 7 A group of people share equally in a \$5,000,000 lottery. Before the money is divided, three more winning ticket holders are declared. As a result, each person's share is reduced by \$375,000. How many people were in the original group of winners?

EXERCISE SET P.8

Practice and Application Exercises

Each day, the number of births in the world exceeds the number of deaths by 229 thousand. The combined number of births and deaths is 521 thousand. Determine the number of births and the number of deaths per day.

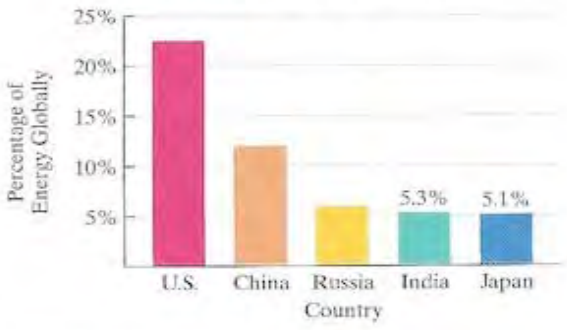
Daily Growth of World Population



Source: "Population Update" 2000

2. The bar graph shows the percentage of global energy used by the countries consuming the most energy. The percentage of global energy used by China exceeds Russia by 6% and the percentage of global energy used by the United States exceeds Russia by 16.4%. Combined, the United States, China, and Russia consume 40.4% of the world's energy. Determine the percentage of global energy used by each country.

Countries Using the Most Energy

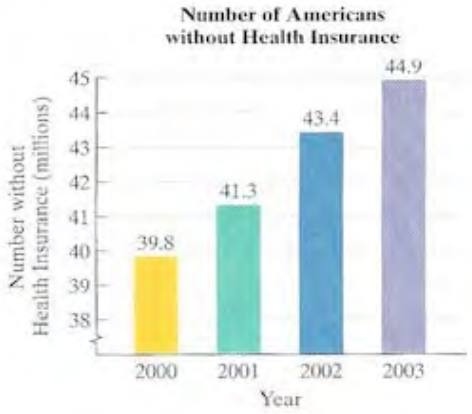


Source: World Bank Group

According to one mathematical model, the average life expectancy for American men born in 1900 was 55 years. Life expectancy has increased by about 0.2 year for each birth year after 1900. Use this information to solve Exercises 3–4.

3. If this trend continues, for which birth year will the average life expectancy be 85 years?
4. If this trend continues, for which birth year will the average life expectancy be 91 years?

The graph shows the number of Americans without health insurance from 2000 through 2003.

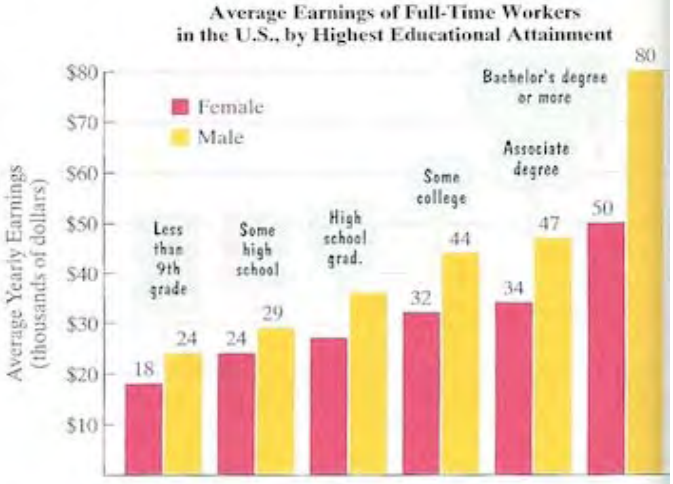


Source: U.S. Census Bureau

In 2000, there were 39.8 million Americans without health insurance. This number has increased at an average rate of 1.7 million people per year. Use this description to solve Exercises 5–6.

5. Determine when the number of Americans without health insurance will exceed the number in 2003 by 8.5 million.
6. Determine when the number of Americans without health insurance will exceed the number in 2003 by 10.2 million.
7. In 2005, there were 13,300 students at college A, with a projected enrollment increase of 1000 students per year. In the same year, there were 26,800 students at college B, with a projected enrollment decline of 500 students per year. According to these projections, when will the colleges have the same enrollment? What will be the enrollment in each college at that time?
8. In 2000, the population of Greece was 10,600,000, with projections of a population decrease of 28,000 people per year. In the same year, the population of Belgium was 10,200,000, with projections of a population decrease of 12,000 people per year. (*Source*: United Nations) According to these projections, when will the two countries have the same population? What will be the population at that time?
9. After a 20% reduction, you purchase a television for $336. What was the television's price before the reduction?
10. After a 30% reduction, you purchase a dictionary for $30.80. What was the dictionary's price before the reduction?
11. Including 8% sales tax, an inn charges $162 per night. Find the inn's nightly cost before the tax is added.
12. Including 5% sales tax, an inn charges $252 per night. Find the inn's nightly cost before the tax is added.

The graph shows average yearly earnings in the United States by highest educational attainment. Use the relevant information shown in the graph to solve Exercises 13–14.



Source: U.S. Census Bureau

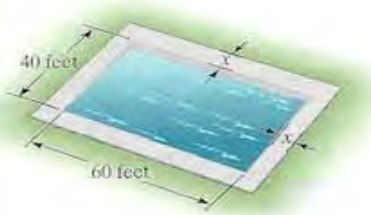
13. The annual salary for men with some college is an increase of 22% over the annual salary for men whose highest educational attainment is a high school degree. What is the annual salary, to the nearest thousand dollars, for men whose highest educational attainment is a high school degree?
14. The annual salary for women with an associate degree is an increase of 26% over the annual salary for women whose highest educational attainment is a high school degree. What is the annual salary, to the nearest thousand dollars, for women whose highest educational attainment is a high school degree?

Exercises 15–16 involve markup, the amount added to the dealer's cost of an item to arrive at the selling price of that item.

15. The selling price of a refrigerator is $584. If the markup is 25% of the dealer's cost, what is the dealer's cost of the refrigerator?
16. The selling price of a scientific calculator is $15. If the markup is 25% of the dealer's cost, what is the dealer's cost of the calculator?
17. A rectangular soccer field is twice as long as it is wide. If the perimeter of the soccer field is 300 yards, what are its dimensions?
18. A rectangular swimming pool is three times as long as it is wide. If the perimeter of the pool is 320 feet, what are its dimensions?
19. The length of the rectangular tennis court at Wimbledon is 6 feet longer than twice the width. If the court's perimeter is 228 feet, what are the court's dimensions?
20. The length of a rectangular pool is 6 meters less than twice the width. If the pool's perimeter is 126 meters, what are its dimensions?

23. The rectangular painting in the figure shown measures 12 inches by 16 inches and contains a frame of uniform width around the four edges. The perimeter of the rectangle formed by the painting and its frame is 72 inches. Determine the width of the frame.

24. The rectangular swimming pool in the figure shown measures 40 feet by 60 feet and contains a path of uniform width around the four edges. The perimeter of the rectangle formed by the pool and the surrounding path is 248 feet. Determine the width of the path.



25. The length of a rectangular sign is 3 feet longer than the width. If the sign's area is 54 square feet, find its length and width.

26. A rectangular parking lot has a length that is 3 yards greater than the width. The area of the parking lot is 180 square yards. Find the length and the width.

27. Each side of a square is lengthened by 3 inches. The area of this new, larger square is 64 square inches. Find the length of a side of the original square.

28. Each side of a square is lengthened by 2 inches. The area of this new, larger square is 36 square inches. Find the length of a side of the original square.

29. A pool measuring 10 meters by 20 meters is surrounded by a path of uniform width. If the area of the pool and the path combined is 600 square meters, what is the width of the path?

30. A vacant rectangular lot is being turned into a community vegetable garden measuring 15 meters by 12 meters. A path of uniform width is to surround the garden. If the area of the lot is 378 square meters, find the width of the path surrounding the garden.

31. As part of a landscaping project, you put in a flower bed measuring 20 feet by 30 feet. To finish off the project, you are putting in a uniform border of pine bark around the outside of the rectangular garden. You have enough pine bark to cover 336 square feet. How wide should the border be?

32. As part of a landscaping project, you put in a flower bed measuring 10 feet by 12 feet. You plan to surround the bed with a uniform border of low-growing plants that require 1 square foot each when mature. If you have 168 of these plants, how wide a strip around the flower bed should you prepare for the border?

31. A 20-foot ladder is 15 feet from a house. How far up the house, to the nearest tenth of a foot, does the ladder reach?

32. The base of a 30-foot ladder is 10 feet from a building. If the ladder reaches the flat roof, how tall, to the nearest tenth of a foot, is the building?

33. A tree is supported by a wire anchored in the ground 5 feet from its base. The wire is 1 foot longer than the height that it reaches on the tree. Find the length of the wire.

34. A tree is supported by a wire anchored in the ground 15 feet from its base. The wire is 4 feet longer than the height that it reaches on the tree. Find the length of the wire.

35. A rectangular piece of land whose length its twice its width has a diagonal distance of 64 yards. How many yards, to the nearest tenth of a yard, does a person save by walking diagonally across the land instead of walking its length and its width?

36. A rectangular piece of land whose length is three times its width has a diagonal distance of 92 yards. How many yards, to the nearest tenth of a yard, does a person save by walking diagonally across the land instead of walking its length and its width?

37. A group of people share equally in a $20,000,000 lottery. Before the money is divided, two more winning ticket holders are declared. As a result, each person's share is reduced by $500,000. How many people were in the original group of winners?

38. A group of friends agrees to equally share the cost of a $480,000 vacation condominium. Before the purchase is made, four more people join the group and enter the agreement. As a result, each person's share is reduced by $32,000. How many people were in the original group?

In Exercises 39–42, use the formula

$$\text{Time traveled} = \frac{\text{Distance traveled}}{\text{Average velocity}}.$$

39. A car can travel 300 miles in the same amount of time it takes a bus to travel 180 miles. If the average velocity of the bus is 20 miles per hour slower than the average velocity of the car, find the average velocity for each.

40. A passenger train can travel 240 miles in the same amount of time it takes a freight train to travel 160 miles. If the average velocity of the freight train is 20 miles per hour slower than the average velocity of the passenger train, find the average velocity of each.

41. You ride your bike to campus a distance of 5 miles and return home on the same route. Going to campus, you ride mostly downhill and average 9 miles per hour faster than on your return trip home. If the round trip takes one hour and ten minutes—that is $\frac{7}{6}$ hours—what is your average velocity on the return trip?

42. An engine pulls a train 140 miles. Then a second engine, whose average velocity is 5 miles per hour faster than the first engine, takes over and pulls the train 200 miles. The total time required for both engines is 9 hours. Find the average velocity of each engine.

43. An automobile repair shop charged a customer \$448, listing \$63 for parts and the remainder for labor. If the cost of labor is \$35 per hour, how many hours of labor did it take to repair the car?

44. A repair bill on a sailboat came to \$1603, including \$532 for parts and the remainder for labor. If the cost of labor is \$63 per hour, how many hours of labor did it take to repair the sailboat?

45. An HMO pamphlet contains the following recommended weight for women: "Give yourself 100 pounds for the first 5 feet plus 5 pounds for every inch over 5 feet tall." Using this description, what height corresponds to a recommended weight of 135 pounds?

46. A job pays an annual salary of \$33,150, which includes a holiday bonus of \$750. If paychecks are issued twice a month, what is the gross amount for each paycheck?

47. You have 35 hits in 140 times at bat. Your batting average is $\frac{35}{140}$, or 0.25. How many consecutive hits must you get to increase your batting average to 0.30?

48. You have 30 hits in 120 times at bat. Your batting average is $\frac{30}{120}$, or 0.25. How many consecutive hits must you get to increase your batting average to 0.28?

Writing in Mathematics

49. In your own words, describe a step-by-step approach for solving algebraic word problems.

50. Write an original word problem that can be solved using an equation. Then solve the problem.

51. The mile records in Example 2 on page 98 are a yardstick for measuring how athletes are getting better and better. Do you think that there is a limit to human performance? Explain your answer. If so, when might we reach it?

52. The bar graph in Exercises 13–14 shows average earnings of U.S. men and women, by highest educational attainment. Describe the trend shown by the graph. Discuss any aspects of the data that surprised you.

53. In your own words, state the Pythagorean Theorem.

54. In the 1939 movie *The Wizard of Oz*, upon being presented with a Th.D. (Doctor of Thinkology), the Scarecrow proudly exclaims, "The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side." Did the Scarecrow get the Pythagorean Theorem right? In particular, describe four errors in the Scarecrow's statement.

Critical Thinking Exercises

55. The perimeter of a plot of land in the shape of a right triangle is 12 miles. If one leg of the triangle exceeds the other leg by 1 mile, find the length of each boundary of the land.

56. The price of a dress is reduced by 40%. When the dress still does not sell, it is reduced by 40% of the reduced price. If the price of the dress after both reductions is \$72, what was the original price?

57. In a film, the actor Charles Coburn plays an elderly "uncle" character criticized for marrying a woman when he is 3 times her age. He wittily replies, "Ah, but in 20 years time I shall only be twice her age." How old is the "uncle" and the woman?

58. Suppose that we agree to pay you 8¢ for every problem in this chapter that you solve correctly and fine you 5¢ for every problem done incorrectly. If at the end of 26 problems we do not owe each other any money, how many problems did you solve correctly?

59. It was wartime when the Ricardos found out Mrs. Ricardo was pregnant. Ricky Ricardo was drafted and made out a will, deciding that \$14,000 in a savings account was to be divided between his wife and his child-to-be. Rather strangely, and certainly with gender bias, Ricky stipulated that if the child were a boy, he would get twice the amount of the mother's portion. If it were a girl, the mother would get twice the amount the girl was to receive. We'll never know what Ricky was thinking of, for (as fate would have it) he did not return from war. Mrs. Ricardo gave birth to twins—a boy and a girl. How was the money divided?

60. A thief steals a number of rare plants from a nursery. On the way out, the thief meets three security guards, one after another. To each security guard, the thief is forced to give one-half the plants that he still has, plus 2 more. Finally, the thief leaves the nursery with 1 lone palm. How many plants were originally stolen?

Group Exercise

61. One of the best ways to learn how to *solve* a word problem in algebra is to *design* word problems of your own. Creating a word problem makes you very aware of precisely how much information is needed to solve the problem. You must also focus on the best way to present information to a reader and on how much information to give. As you write your problem, you gain skills that will help you solve problems created by others.

The group should design five different word problems that can be solved using equations. All of the problems should be on different topics. For example, the group should not have more than one problem on the perimeter of a rectangle. The group should turn in both the problems and their algebraic solutions.

Step 4 Solve the inequality and answer the question.

$$4 + 0.15x < 20 + 0.05x$$ This is the inequality that models the verbal conditions.

$$4 + 0.15x - 0.05x < 20 + 0.05x - 0.05x$$ Subtract 0.05x from both sides.

$$4 + 0.1x < 20$$ Simplify.

$$4 + 0.1x - 4 < 20 - 4$$ Subtract 4 from both sides.

$$0.1x < 16$$ Simplify.

$$\frac{0.1x}{0.1} < \frac{16}{0.1}$$ Divide both sides by 0.1.

$$x < 160$$ Simplify.

Thus, driving fewer than 160 miles per day makes Acme the better deal.

Step 5 Check the proposed solution in the original wording of the problem. One way to do this is to take a mileage less than 160 miles per day to see if Acme is the better deal. Suppose that 150 miles are driven in a day.

$$\text{Cost for Acme} = 4 + 0.15(150) = 26.50$$
$$\text{Cost for Interstate} = 20 + 0.05(150) = 27.50$$

Acme has a lower daily cost, making Acme the better deal.

Check Point 9 A car can be rented from Basic Rental for $260 per week with no extra charge for mileage. Continental charges $80 per week plus 25 cents for each mile driven to rent the same car. How many miles must be driven in a week to make the rental cost for Basic Rental a better deal than Continental's?

EXERCISE SET P.9

Practice Exercises

In Exercises 1–14, express each interval in set-builder notation and graph the interval on a number line.

1. $(1, 6]$ **2.** $(-2, 4]$
3. $[-5, 2)$ **4.** $[-4, 3)$
5. $[-3, 1]$ **6.** $[-2, 5]$
7. $(2, \infty)$ **8.** $(3, \infty)$
9. $[-3, \infty)$ **10.** $[-5, \infty)$
11. $(-\infty, 3)$ **12.** $(-\infty, 2)$
13. $(-\infty, 5.5)$ **14.** $(-\infty, 3.5]$

In Exercises 15–26, use graphs to find each set.

15. $(-3, 0) \cap [-1, 2]$ **16.** $(-4, 0) \cap [-2, 1]$
17. $(-3, 0) \cup [-1, 2]$ **18.** $(-4, 0) \cup [-2, 1]$
19. $(-\infty, 5) \cap [1, 8)$ **20.** $(-\infty, 6) \cap [2, 9)$
21. $(-\infty, 5) \cup [1, 8)$ **22.** $(-\infty, 6) \cup [2, 9)$
23. $[3, \infty) \cap (6, \infty)$ **24.** $[2, \infty) \cap (4, \infty)$
25. $[3, \infty) \cup (6, \infty)$ **26.** $[2, \infty) \cup (4, \infty)$

In all exercises, use interval notation to express solution sets and graph each solution set on a number line.

In Exercises 27–48, solve each linear inequality.

27. $5x + 11 < 26$ **28.** $2x + 5 < 17$
29. $3x - 7 \geq 13$ **30.** $8x - 2 \geq 14$
31. $-9x \geq 36$ **32.** $-5x \leq 30$
33. $8x - 11 \leq 3x - 13$ **34.** $18x + 45 \leq 12x - 8$
35. $4(x + 1) + 2 \geq 3x + 6$
36. $8x + 3 > 3(2x + 1) + x + 5$
37. $2x - 11 < -3(x + 2)$ **38.** $-4(x + 2) > 3x + 20$
39. $1 - (x + 3) \geq 4 - 2x$ **40.** $5(3 - x) \leq 3x - 1$
41. $\frac{x}{4} - \frac{3}{2} \leq \frac{x}{2} + 1$ **42.** $\frac{3x}{10} + 1 \geq \frac{1}{5} - \frac{x}{10}$
43. $1 - \frac{x}{2} > 4$ **44.** $7 - \frac{4}{5}x < \frac{3}{5}$
45. $\frac{x - 4}{6} \geq \frac{x - 2}{9} + \frac{5}{18}$ **46.** $\frac{4x - 3}{6} + 2 \geq \frac{2x - 1}{12}$
47. $3[3(x + 5) + 8x + 7] + 5[3(x - 6) - 2(3x - 5)] < 2(4x + 3)$
48. $5[3(2 - 3x) - 2(5 - x)] - 6[5(x - 2) - 2(4x - 3)] < 3x + 19$

In Exercises 49–56, solve each compound inequality.

49. $6 < x + 3 < 8$ **50.** $7 < x + 5 < 11$
51. $-3 \leq x - 2 < 1$ **52.** $-6 < x - 4 \leq 1$
53. $-11 < 2x - 1 \leq -5$ **54.** $3 \leq 4x - 3 < 19$
55. $-3 \leq \frac{2}{3}x - 5 < -1$ **56.** $-6 \leq \frac{1}{2}x - 4 < -3$

In Exercises 57–92, solve each absolute value inequality.

57. $|x| < 3$ **58.** $|x| < 5$
59. $|x - 1| \leq 2$ **60.** $|x + 3| \leq 4$
61. $|2x - 6| < 8$ **62.** $|3x + 5| < 17$
63. $|2(x - 1) + 4| \leq 8$ **64.** $|3(x - 1) + 2| \leq 20$

65. $\left|\frac{2x+6}{3}\right| < 2$

66. $\left|\frac{3(x-1)}{4}\right| < 6$

67. $|x| > 3$

68. $|x| > 5$

69. $|x-1| \geq 2$

70. $|x+3| \geq 4$

71. $|3x-8| > 7$

72. $|5x-2| > 13$

73. $\left|\frac{2x+2}{4}\right| \geq 2$

74. $\left|\frac{3x-3}{9}\right| \geq 1$

75. $\left|3-\frac{2}{3}x\right| > 5$

76. $\left|3-\frac{3}{4}x\right| > 9$

77. $3|x-1|+2 \geq 8$

78. $5|2x+1|-3 \geq 9$

79. $-2|x-4| \geq -4$

80. $-3|x+7| \geq -27$

81. $-4|1-x| < -16$

82. $-2|5-x| < -6$

83. $3 \leq |2x-1|$

84. $9 \leq |4x+7|$

85. $5 > |4-x|$

86. $2 > |11-x|$

87. $1 < |2-3x|$

88. $4 < |2-x|$

89. $12 < \left|-2x+\frac{6}{7}\right| + \frac{3}{7}$

90. $1 < \left|x-\frac{11}{3}\right| + \frac{7}{3}$

91. $4 + \left|3-\frac{x}{3}\right| \geq 9$

92. $\left|2-\frac{x}{2}\right| - 1 \leq 1$

Practice Plus

In Exercises 93–96, use interval notation to represent all values of x satisfying the given conditions.

93. $y = 1-(x+3)+2x$ and y is at least 4.

94. $y = 2x-11+3(x+2)$ and y is at most 0.

95. $y = 7-\left|\frac{x}{2}+2\right|$ and y is at most 4.

96. $y = 8-|5x+3|$ and y is at least 6.

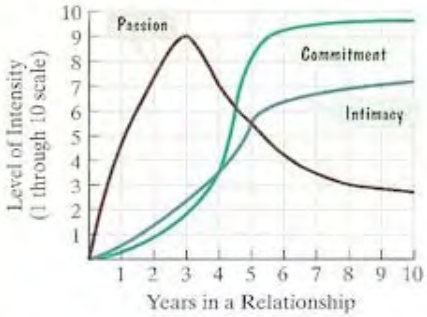
97. When 3 times a number is subtracted from 4, the absolute value of the difference is at least 5. Use interval notation to express the set of all numbers that satisfy this condition.

98. When 4 times a number is subtracted from 5, the absolute value of the difference is at most 13. Use interval notation to express the set of all numbers that satisfy this condition.

Application Exercises

The graphs show that the three components of love, namely passion, intimacy, and commitment, progress differently over time. Passion peaks early in a relationship and then declines. By contrast, intimacy and commitment build gradually. Use the graphs to solve Exercises 99–106.

The Course of Love Over Time



Source: R. J. Sternberg. A Triangular Theory of Love, *Psychological Review*, 93, 119–135.

99. Use interval notation to write an inequality that expresses for which years in a relationship intimacy is greater than commitment.

100. Use interval notation to write an inequality that expresses for which years in a relationship passion is greater than or equal to intimacy.

101. What is the relationship between passion and intimacy on for years $[5, 7)$?

102. What is the relationship between intimacy and commitment for years $[4, 7)$?

103. What is the relationship between passion and commitment for years $(6, 8)$?

104. What is the relationship between passion and commitment for years $(7, 9)$?

105. What is the maximum level of intensity for passion? After how many years in a relationship does this occur?

106. After approximately how many years do levels of intensity for commitment exceed the maximum level of intensity for passion?

107. The percentage, P, of U.S. voters who used electronic voting systems, such as optical scans, in national elections can be modeled by the formula

$$P = 3.1x + 25.8,$$

where x is the number of years after 1994. In which years will more than 63% of U.S. voters use electronic systems?

108. The percentage, P, of U.S. voters who used punch cards or lever machines in national elections can be modeled by the formula

$$P = -2.5x + 63.1,$$

where x is the number of years after 1994. In which years will fewer than 38.1% of U.S. voters use punch cards or lever machines?

109. A basic cellular phone plan costs \$20 per month for 60 calling minutes. Additional time costs \$0.40 per minute. The formula

$$C = 20 + 0.40(x - 60)$$

gives the monthly cost for this plan, C, for x calling minutes, where $x > 60$. How many calling minutes are possible for a monthly cost of at least \$28 and at most \$40?

110. The formula for converting Fahrenheit temperature, F, to Celsius temperature, C, is

$$C = \frac{5}{9}(F - 32).$$

If Celsius temperature ranges from 15° to 35°, inclusive, what is the range for the Fahrenheit temperature? Use interval notation to express this range.

111. If a coin is tossed 100 times, we would expect approximately 50 of the outcomes to be heads. It can be demonstrated that a coin is unfair if h, the number of outcomes that result in heads, satisfies $\left|\frac{h-50}{5}\right| \geq 1.645$. Describe the number of outcomes that determine an unfair coin that is tossed 100 times.

In Exercises 112–123, use the strategy for solving word problems, translating from the verbal conditions of the problem to a linear inequality.

112. A truck can be rented from Basic Rental for \$50 per day plus \$0.20 per mile. Continental charges \$20 per day plus \$0.50 per mile to rent the same truck. How many miles must be driven in a day to make the rental cost for Basic Rental a better deal than Continental's?

113. You are choosing between two long-distance telephone plans. Plan A has a monthly fee of \$15 with a charge of \$0.08 per minute for all long-distance calls. Plan B has a monthly fee of \$3 with a charge of \$0.12 per minute for all long-distance calls. How many minutes of long-distance calls in a month make plan A the better deal?

114. A city commission has proposed two tax bills. The first bill requires that a homeowner pay \$1800 plus 3% of the assessed home value in taxes. The second bill requires taxes of \$200 plus 8% of the assessed home value. What price range of home assessment would make the first bill a better deal?

115. A local bank charges \$8 per month plus 5¢ per check. The credit union charges \$2 per month plus 8¢ per check. How many checks should be written each month to make the credit union a better deal?

116. A company manufactures and sells blank audiocassette tapes. The weekly fixed cost is \$10,000 and it costs \$0.40 to produce each tape. The selling price is \$2.00 per tape. How many tapes must be produced and sold each week for the company to generate a profit?

117. A company manufactures and sells personalized stationery. The weekly fixed cost is \$3000 and it costs \$3.00 to produce each package of stationery. The selling price is \$5.50 per package. How many packages of stationery must be produced and sold each week for the company to generate a profit?

118. An elevator at a construction site has a maximum capacity of 2800 pounds. If the elevator operator weighs 265 pounds and each cement bag weighs 65 pounds, how many bags of cement can be safely lifted on the elevator in one trip?

119. An elevator at a construction site has a maximum capacity of 3000 pounds. If the elevator operator weighs 245 pounds and each cement bag weighs 95 pounds, how many bags of cement can be safely lifted on the elevator in one trip?

120. To earn an A in a course, you must have a final average of at least 90%. On the first four examinations, you have grades of 86%, 88%, 92%, and 84%. If the final examination counts as two grades, what must you get on the final to earn an A in the course?

121. On two examinations, you have grades of 86 and 88. There is an optional final examination, which counts as one grade. You decide to take the final in order to get a course grade of A, meaning a final average of at least 90.

a. What must you get on the final to earn an A in the course?

b. By taking the final, if you do poorly, you might risk the B that you have in the course based on the first two exam grades. If your final average is less than 80, you will lose your B in the course. Describe the grades on the final that will cause this to happen.

122. Parts for an automobile repair cost \$175. The mechanic charges \$34 per hour. If you receive an estimate for at least \$226 and at most \$294 for fixing the car, what is the time interval that the mechanic will be working on the job?

123. The toll to a bridge is \$3.00. A three-month pass costs \$7.50 and reduces the toll to \$0.50. A six-month pass costs \$30 and permits crossing the bridge for no additional fee. How many crossings per three-month period does it take for the three-month pass to be the best deal?

Writing in Mathematics

124. When graphing the solutions of an inequality, what does a parenthesis signify? What does a bracket signify?

125. Describe ways in which solving a linear inequality is similar to solving a linear equation.

126. Describe ways in which solving a linear inequality is different than solving a linear equation.

127. What is a compound inequality and how is it solved?

128. Describe how to solve an absolute value inequality involving the symbol $<$. Give an example.

129. Describe how to solve an absolute value inequality involving the symbol $>$. Give an example.

130. Explain why $|x| < -4$ has no solution.

131. Describe the solution set of $|x| > -4$.

Critical Thinking Exercises

132. Which one of the following is true?

a. The first step in solving $|2x - 3| > -7$ is to rewrite the inequality as $2x - 3 > -7$ or $2x - 3 < 7$.

b. The smallest real number in the solution set of $2x > 6$ is 4.

c. All irrational numbers satisfy $|x - 4| > 0$.

d. None of these statements is true.

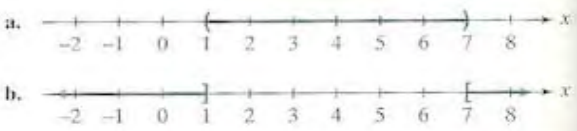
133. What's wrong with this argument? Suppose x and y represent two real numbers, where $x > y$.

$2 > 1$	This is a true statement.
$2(y - x) > 1(y - x)$	Multiply both sides by $y - x$.
$2y - 2x > y - x$	Use the distributive property.
$y - 2x > -x$	Subtract y from both sides.
$y > x$	Add $2x$ to both sides.

The final inequality, $y > x$, is impossible because we were initially given $x > y$.

134. Write an absolute value inequality for which the interval shown is the solution.

Solutions lie within 3 units of 4.



135. Here are two inequalities that describe the range of monthly average temperatures, T, in degrees Fahrenheit for two American cities:

Model 1: $|T - 57| < 7$

Model 2: $|T - 50| < 22$.

Which model describes Albany, New York, and which model describes San Francisco, California?

Group Exercise

136. Each group member should research one situation that provides two different pricing options. These can involve areas such as public transportation options (with or without coupon books), cell phone plans, long-distance telephone plans, or anything of interest. Be sure to bring in all the details for each option. At a second group meeting, select the two pricing situations that are most interesting and relevant. Using each situation, write a word problem about selecting the better of the two options. The word problem should be one that can be solved using a linear inequality. The group should turn in the two problems and their solutions.

8. Consider the set:

$$\left\{-17, -\frac{9}{13}, 0, 0.75, \sqrt{2}, \pi, \sqrt{81}\right\}.$$

List all numbers from the set that are **a.** natural numbers, **b.** whole numbers, **c.** integers, **d.** rational numbers, **e.** irrational numbers, **f.** real numbers.

In Exercises 9–11, rewrite each expression without absolute value bars.

9. $|-103|$ **10.** $|\sqrt{2} - 1|$

11. $|3 - \sqrt{17}|$

12. Express the distance between the numbers -17 and 4 using absolute value. Then evaluate the absolute value.

In Exercises 13–18, state the name of the property illustrated.

13. $3 + 17 = 17 + 3$

14. $(6 \cdot 3) \cdot 9 = 6 \cdot (3 \cdot 9)$

15. $\sqrt{3}(\sqrt{5} + \sqrt{3}) = \sqrt{15} + 3$

16. $(6 \cdot 9) \cdot 2 = 2 \cdot (6 \cdot 9)$

17. $\sqrt{3}(\sqrt{5} + \sqrt{3}) = (\sqrt{5} + \sqrt{3})\sqrt{3}$

18. $(3 \cdot 7) + (4 \cdot 7) = (4 \cdot 7) + (3 \cdot 7)$

In Exercises 19–22, simplify each algebraic expression.

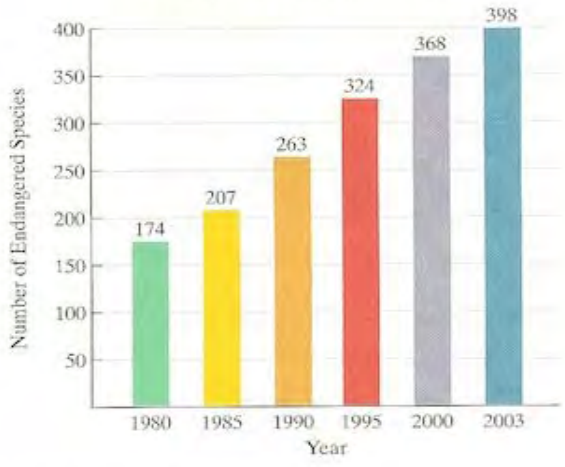
19. $5(2x - 3) + 7x$

20. $\frac{1}{5}(5x) + [(3y) + (-3y)] - (-x)$

21. $3(4y - 5) - (7y + 2)$ **22.** $8 - 2[3 - (5x - 1)]$

23. The bar graph shows the number of endangered animal species in the United States for six selected years. The data can be modeled by the formulas $E = 10x + 166$ and $E = 0.04x^2 + 9.2x + 169$, in which E represents the number of endangered species x years after 1980. Which formula best describes the actual number of endangered animal species in 2000?

Endangered Animal Species in the U.S.



Source: U.S. Fish and Wildlife Service

P.2

Evaluate each exponential expression in Exercises 24–27.

24. $(-3)^3(-2)^2$ **25.** $2^{-4} + 4^{-1}$

26. $5^{-3} \cdot 5$ **27.** $\frac{3^3}{3^6}$

Simplify each exponential expression in Exercises 28–31.

28. $(-2x^4y^3)^3$ **29.** $(-5x^3y^2)(-2x^{-11}y^{-2})$

30. $(2x^3)^{-4}$ **31.** $\frac{7x^5y^6}{28x^{15}y^{-2}}$

In Exercises 32–33, write each number in decimal notation.

32. 3.74×10^4 **33.** 7.45×10^{-5}

In Exercises 34–35, write each number in scientific notation.

34. 3,590,000 **35.** 0.00725

In Exercises 36–37, perform the indicated operation and write the answer in decimal notation.

36. $(3 \times 10^3)(1.3 \times 10^2)$ **37.** $\frac{6.9 \times 10^3}{3 \times 10^5}$

38. If you earned \$1 million per year ($\$10^6$), how long would it take to accumulate \$1 billion ($\$10^9$)?

39. If the population of the United States is 2.9×10^8 and each person spends about \$150 per year going to the movies (or renting movies), express the total annual spending on movies in scientific notation.

P.3

Use the product rule to simplify the expressions in Exercises 40–43. In Exercises 42–43, assume that variables represent nonnegative real numbers.

40. $\sqrt{300}$ **41.** $\sqrt{12x^2}$

42. $\sqrt{10x} \cdot \sqrt{2x}$ **43.** $\sqrt{r^3}$

Use the quotient rule to simplify the expressions in Exercises 44–45.

44. $\sqrt{\frac{121}{4}}$ **45.** $\frac{\sqrt{96x^3}}{\sqrt{2x}}$ (Assume that $x > 0$.)

In Exercises 46–48, add or subtract terms whenever possible.

46. $7\sqrt{5} + 13\sqrt{5}$ **47.** $2\sqrt{50} + 3\sqrt{8}$

48. $4\sqrt{72} - 2\sqrt{48}$

In Exercises 49–52, rationalize the denominator.

49. $\frac{30}{\sqrt{5}}$ **50.** $\frac{\sqrt{2}}{\sqrt{3}}$

51. $\frac{5}{6 + \sqrt{3}}$ **52.** $\frac{14}{\sqrt{7} - \sqrt{5}}$

Evaluate each expression in Exercises 53–56 or indicate that the root is not a real number.

53. $\sqrt[3]{125}$ **54.** $\sqrt[5]{-32}$

55. $\sqrt[4]{-125}$ **56.** $\sqrt[4]{(-5)^4}$

Simplify the radical expressions in Exercises 57–61.

57. $\sqrt[3]{81}$ **58.** $\sqrt[3]{y^5}$

59. $\sqrt[4]{8} \cdot \sqrt[4]{10}$ **60.** $4\sqrt[3]{16} + 5\sqrt[3]{2}$

61. $\dfrac{\sqrt[4]{32x^5}}{\sqrt[4]{16x}}$ (Assume that $x > 0$.)

n Exercises 62–67, evaluate each expression.

62. $16^{\frac{1}{2}}$ **63.** $25^{-\frac{1}{2}}$

64. $125^{\frac{1}{3}}$ **65.** $27^{-\frac{1}{3}}$

66. $64^{\frac{2}{3}}$ **67.** $27^{-\frac{4}{3}}$

n Exercises 68–70, simplify using properties of exponents.

68. $\left(5x^{\frac{2}{3}}\right)\left(4x^{\frac{1}{4}}\right)$ **69.** $\dfrac{15x^{\frac{3}{4}}}{5x^{\frac{1}{2}}}$

70. $(125x^6)^{\frac{2}{3}}$

71. Simplify by reducing the index of the radical: $\sqrt[6]{y^3}$.

.4

n Exercises 72–73, perform the indicated operations. Write the esulting polynomial in standard form and indicate its degree.

72. $(-6x^3 + 7x^2 - 9x + 3) + (14x^3 + 3x^2 - 11x - 7)$

73. $(13x^4 - 8x^3 + 2x^2) - (5x^4 - 3x^3 + 2x^2 - 6)$

n Exercises 74–80, find each product.

74. $(3x - 2)(4x^2 + 3x - 5)$ **75.** $(3x - 5)(2x + 1)$

76. $(4x + 5)(4x - 5)$ **77.** $(2x + 5)^2$

78. $(3x - 4)^2$ **79.** $(2x + 1)^3$

80. $(5x - 2)^3$

n Exercises 81–87, find each product.

81. $(x + 7y)(3x - 5y)$ **82.** $(3x - 5y)^2$

83. $(3x^2 + 2y)^2$ **84.** $(7x + 4y)(7x - 4y)$

85. $(a - b)(a^2 + ab + b^2)$

86. $[5y - (2x + 1)][5y + (2x + 1)]$

87. $(x + 2y + 4)^2$

.5

n Exercises 88–104, factor completely, or state that the polynomial prime.

88. $15x^3 + 3x^2$ **89.** $x^2 - 11x + 28$

90. $15x^2 - x - 2$ **91.** $64 - x^2$

92. $x^2 + 16$ **93.** $3x^4 - 9x^3 - 30x^2$

94. $20x^7 - 36x^3$ **95.** $x^3 - 3x^2 - 9x + 27$

96. $16x^2 - 40x + 25$ **97.** $x^4 - 16$

98. $y^3 - 8$ **99.** $x^3 + 64$

100. $3x^4 - 12x^2$ **101.** $27x^3 - 125$

102. $x^5 - x$ **103.** $x^3 + 5x^2 - 2x - 10$

104. $x^2 + 18x + 81 - y^2$

Exercises 105–107, factor and simplify each algebraic pression.

105. $16x^{-\frac{3}{4}} + 32x^{\frac{1}{4}}$

106. $(x^2 - 4)(x^2 + 3)^{\frac{1}{2}} - (x^2 - 4)^2(x^2 + 3)^{\frac{3}{2}}$

107. $12x^{-\frac{1}{2}} + 6x^{-\frac{3}{2}}$

P.6

In Exercises 108–110, simplify each rational expression. Also, list all numbers that must be excluded from the domain.

108. $\dfrac{x^3 + 2x^2}{x + 2}$ **109.** $\dfrac{x^2 + 3x - 18}{x^2 - 36}$

110. $\dfrac{x^2 + 2x}{x^2 + 4x + 4}$

In Exercises 111–113, multiply or divide as indicated.

111. $\dfrac{x^2 + 6x + 9}{x^2 - 4} \cdot \dfrac{x + 3}{x - 2}$ **112.** $\dfrac{6x + 2}{x^2 - 1} \div \dfrac{3x^2 + x}{x - 1}$

113. $\dfrac{x^2 - 5x - 24}{x^2 - x - 12} \div \dfrac{x^2 - 10x + 16}{x^2 + x - 6}$

In Exercises 114–117, add or subtract as indicated.

114. $\dfrac{2x - 7}{x^2 - 9} - \dfrac{x - 10}{x^2 - 9}$ **115.** $\dfrac{3x}{x + 2} + \dfrac{x}{x - 2}$

116. $\dfrac{x}{x^2 - 9} + \dfrac{x - 1}{x^2 - 5x + 6}$ **117.** $\dfrac{4x - 1}{2x^2 + 5x - 3} - \dfrac{x + 3}{6x^2 + x - 2}$

In Exercises 118–120, simplify each expression.

118. $\dfrac{\frac{1}{x} - \frac{1}{2}}{\frac{1}{3} - \frac{x}{6}}$ **119.** $\dfrac{3 + \frac{12}{x}}{1 - \frac{16}{x^2}}$ **120.** $\dfrac{3 - \frac{1}{x + 3}}{3 + \frac{1}{x + 3}}$

121. $\dfrac{\sqrt{25 - x^2} + \frac{x^2}{\sqrt{25 - x^2}}}{25 - x^2}$

P.7

In Exercises 122–135, solve each equation.

122. $1 - 2(6 - x) = 3x + 2$

123. $2(x - 4) + 3(x + 5) = 2x - 2$

124. $2x - 4(5x + 1) = 3x + 17$

125. $\dfrac{1}{x - 1} - \dfrac{1}{x + 1} = \dfrac{2}{x^2 - 1}$

126. $\dfrac{4}{x + 2} + \dfrac{2}{x - 4} = \dfrac{30}{x^2 - 2x - 8}$

127. $-4|2x + 1| + 12 = 0$ **128.** $2x^2 - 11x + 5 = 0$

129. $(3x + 5)(x - 3) = 5$ **130.** $3x^2 - 7x + 1 = 0$

131. $x^2 - 9 = 0$ **132.** $(x - 3)^2 - 24 = 0$

133. $\dfrac{2x}{x^2 + 6x + 8} = \dfrac{x}{x + 4} - \dfrac{2}{x + 2}$

134. $\sqrt{8 - 2x} - x = 0$ **135.** $\sqrt{2x - 3} + x = 3$

In Exercises 136–137, solve each formula for the specified variable.

136. $vt + gt^2 = s$ for g **137.** $T = \dfrac{A - P}{Pr}$ for P

In Exercises 138–139, without solving the given quadratic equation, determine the number and type of solutions.

138. $x^2 = 2x - 19$ **139.** $9x^2 - 30x + 25 = 0$

P.8

In Exercises 140–149, use the five-step strategy for solving word problems.

140. The fast-food chains may be touting their "new and improved" salads, but how do they measure up in terms of calories?

Burger King
Chicken Caesar

Taco Bell
Express Taco Salad

Number of calories exceeds the Chicken Caesar by 125.

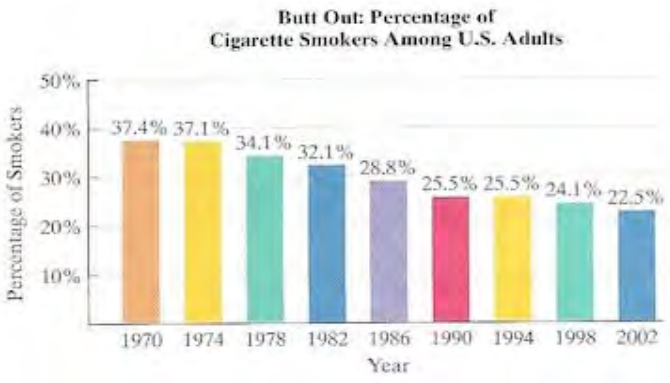
Wendy's
Mandarin Chicken Salad

Number of calories exceeds the Chicken Caesar by 95.

Source: Newsweek

Combined, the three salads contain 1705 calories. Determine the number of calories in each salad.

141. The bar graph shows that in 1970, 37.4% of U.S. adults smoked cigarettes. For the period from 1970 through 2002, the percentage of smokers among U.S. adults decreased at an average rate of 0.5% per year. If this trend continues, when will only 18.4% of U.S. adults smoke cigarettes?



Source: Centers for Disease Control and Prevention

142. After a 20% price reduction, a cordless phone sold for $48. What was the phone's price before the reduction?

143. A salesperson earns $300 per week plus 5% commission of sales. How much must be sold to earn $800 in a week?

144. The length of a rectangular field is 6 yards less than triple the width. If the perimeter of the field is 340 yards, what are its dimensions?

145. In 2007, there were 14,100 students at college A, with a projected enrollment increase of 1500 students per year. In the same year, there were 41,700 students at college B, with a projected enrollment decline of 800 students per year. In which year will the colleges have the same enrollment? What will be the enrollment in each college at that time?

146 An architect is allowed 15 square yards of floor space to add a small bedroom to a house. Because of the room's design in relationship to the existing structure, the width of the rectangular floor must be 7 yards less than two times the length. Find the length and width of the rectangular floor that the architect is permitted.

147. A building casts a shadow that is double the length of its height. If the distance from the end of the shadow to the top of the building is 300 meters, how high is the building? Round to the nearest meter.

148. A painting measuring 10 inches by 16 inches is surrounded by a frame of uniform width. If the combined area of the painting and frame is 280 square inches, determine the width of the frame.

149. Club members equally share the cost of $1500 to charter a fishing boat. Shortly before the boat is to leave, four people decide not to go due to rough seas. As a result, the cost per person is increased by $100. How many people originally intended to go on the fishing trip?

P.9

In Exercises 150–152, express each interval in set-builder notation and graph the interval on a number line.

150. $[-3, 5)$ **151.** $(-2, \infty)$ **152.** $(-\infty, 0]$

In Exercises 153–156, use graphs to find each set.

153. $(-2, 1] \cap [-1, 3)$ **154.** $(-2, 1] \cup [-1, 3)$

155. $[1, 3) \cap (0, 4)$ **156.** $[1, 3) \cup (0, 4)$

In Exercises 157–166, solve each inequality. Use interval notation to express solution sets and graph each solution set on a number line.

157. $-6x + 3 \le 15$ **158.** $6x - 9 \ge -4x - 3$

159. $\frac{x}{3} - \frac{3}{4} - 1 > \frac{x}{2}$ **160.** $6x + 5 > -2(x - 3) - 25$

161. $3(2x - 1) - 2(x - 4) \ge 7 + 2(3 + 4x)$

162. $7 < 2x + 3 \le 9$ **163.** $|2x + 3| \le 15$

164. $\left|\frac{2x + 6}{3}\right| > 2$ **165.** $|2x + 5| - 7 \ge -6$

166. $-4|x + 2| + 5 \le -7$

167. A car rental agency rents a certain car for $40 per day with unlimited mileage or $24 per day plus $0.20 per mile. How far can a customer drive this car per day for the $24 option to cost no more than the unlimited mileage option?

168. To receive a B in a course, you must have an average of at least 80% but less than 90% on five exams. Your grades on the first four exams were 95%, 79%, 91%, and 86%. What range of grades on the fifth exam will result in a B for the course?